現代宇宙論

No. 4

ビッグバン宇宙論の概要

- ☆ ハッブルの法則（と一般相対論による予言）が示唆するように，宇宙は膨張している
- ☆ 過去の宇宙は小さかったはず
- ☆ 過去には物質やエネルギーの密度が大きかったはず
- ☆ 物質の放射や温度も高かったはず
- ☆ 宇宙は，高温高密度の火の玉状態からスタートしたと考えられる
- ☆ 膨張とともに密度が希薄になり，温度も下がった

ビッグバン宇宙論の歴史

- ⭐ 1916年，アインシュタイン方程式（宇宙項なし版）
- ⭐ 1917年，アインシュタイン方程式（宇宙項入り）
- ⭐ 1922年，フリードマンが宇宙項なしのアインシュタイン方程式を解いて，膨張宇宙解を発見
- ⭐ 1927年，ルメートルが宇宙項ありのアインシュタイン方程式を解く
- ⭐ 1929年，ハッブルの法則

ビッグバン宇宙論の歴史

- ジョージ・ガモフが1948年にビッグバン宇宙論を提案（αβγ理論）
- ビッグバン宇宙論の観測的証拠はまだ少なかった
- 定常宇宙論(水素原子が1個/cm^3/50億年の割合で作り出される。1948年にフレッド・ホイルが提唱)と膨張宇宙論が拮抗
- 「ビッグバン」はフレッド・ホイルが膨張宇宙論を揶揄してつけた名称
- 結果的には，定常宇宙論は観測的に棄却され，ビッグバンが残った

αβγ理論の原論文

PHYSICAL REVIEW VOLUME 73, NUMBER 7 APRIL 1, 1948

Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.

The Origin of Chemical Elements

R. A. Alpher* α
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland

AND

H. Bethe β
Cornell University, Ithaca, New York

AND

G. Gamow γ
The George Washington University, Washington, D. C.

February 18, 1948

As pointed out by one of us,[1] various nuclear species must have originated not as the result of an equilibrium corresponding to a certain temperature and density,

We may remark at first that the building-up process was apparently completed when the temperature of the neutron gas was still rather high, since otherwise the observed abundances would have been strongly affected by the resonances in the region of the slow neutrons. According to Hughes,[2] the neutron capture cross sections of various elements (for neutron energies of about 1 Mev) increase exponentially with atomic number halfway up the periodic system, remaining approximately constant for heavier elements.

Using these cross sections, one finds by integrating Eqs. (1) as shown in Fig. 1 that the relative abundances of various nuclear species decrease rapidly for the lighter elements and remain approximately constant for the elements heavier than silver. In order to fit the calculated curve with the observed abundances[3] it is necessary to assume the integral of $\rho_n dt$ during the building-up period is equal to 5×10^4 g sec./cm^3.

On the other hand, according to the relativistic theory of the expanding universe[4] the density dependence on time is given by $\rho \cong 10^6/t^2$. Since the integral of this expression diverges at $t=0$, it is necessary to assume that the building-up process began at a certain time t_0, satisfying the relation:

$$\int_{t_0}^{\infty} (10^6/t^2)dt \cong 5\times10^4, \qquad (2)$$

The paper by Gamow and Alpher is generally known as the "$\alpha\beta\gamma$ paper." Quite apart from its content, the paper has a famous history, which has made it part of the physics folklore. As it appeared in *Physical Review*, it was co-authored by Bethe, although he did not, in fact, contribute at all. Gamow recalled the story behind the $\alpha\beta\gamma$ terminology as follows: "In writing up the preliminary communication of this work, I was unhappy that the letter β was missing between α and γ. Thus, sending the manuscript for publication in *Phys. Rev.*, I put in the name of Hans Bethe (in absentia) between our names. This was planned as a surprise to Hans when he would unexpectedly find his name as co-author and I was sure that, being my old friend, and having a good sense of humor he would not mind. What I did not know was that at that time he was one of the reviewers for *Phys. Rev.* and that the manuscript was sent to him for evaluation. But he did not make any changes in it except to strike out the words "in absentia" after his name, thus endorsing the idea and the results."[94] Gamow even tried to make Herman change his name to Delter so the $\alpha\beta\gamma$ theory would become an $\alpha\beta\gamma\delta$ theory. Although Herman resisted the temptation, in a later paper Gamow referred to "the neutron-capture theory of the origin of atomic species recently developed by Alpher, Bethe, Gamow and Delter," a reference which must have caused some confusion.[95] Bethe, who was not foreign to this kind of fun, took the joke with good humor as Gamow expected. As he later told Alpher and Herman, "I felt at the time that it was rather a nice joke, and that the paper had a chance of being correct, so that I did not mind my name being added to it."[96] The last word on the matter was added by Gamow in 1960: "There was . . . a rumor that later, when the $\alpha\beta\gamma$ theory went temporarily on the rocks, Dr. Bethe seriously considered changing his name to Zacharias."[97]

H.Kragh, "Cosmology and Controversy"

フリードマン方程式

ロバートソンウォーカー計量

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega\right)$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

時刻tのハッブル定数

ただし$c = 1$となる単位系をとってある

フリードマン方程式

$$\frac{\dot{a}^2}{a^2} + \frac{K}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2} - \Lambda = -8\pi G p$$

フリードマン方程式を解いて宇宙の振る舞いを調べる

臨界密度

$$\frac{\dot{a}^2}{a^2} + \frac{K}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho \text{より} \frac{K}{a^2} = \frac{8\pi G}{3}(\rho + \rho_\Lambda) - H^2$$

ただし $\rho_\Lambda = \dfrac{\Lambda}{8\pi G}$

↑ まとめてエネルギー密度扱いする

$\rho_c = \dfrac{3H^2}{8\pi G}$ (臨界密度) として

<u>任意の時点で</u>

$$\begin{cases} \rho + \rho_\Lambda > \rho_c & K > 0\text{: 閉じた宇宙} \\ \rho + \rho_\Lambda = \rho_c & K = 0\text{: 平坦な宇宙} \\ \rho + \rho_\Lambda < \rho_c & K < 0\text{: 開いた宇宙} \end{cases}$$

これは宇宙の組成に関係なく，現在の密度値で決まる！

現在の値: $\rho_{c,0} = \dfrac{3H_0^2}{8\pi G} \simeq 1.878 \times 10^{-29} h^2 \mathrm{g/cm^3}$

ハッブルパラメータ: $h = \dfrac{H_0}{100\ \mathrm{km/s/Mpc}} \simeq 0.67$

フリードマン方程式を解く

先ほどのように変形すると，宇宙項をあつかいやすくなる

$$\frac{\dot{a}^2}{a^2} + \frac{K}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho$$

$$\rho = \rho_{\mathrm{mat}} + \rho_{\mathrm{rad}}$$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}$$

とすると

$$\frac{\dot{a}^2}{a^2} + \frac{K}{a^2} = \frac{8\pi G}{3}(\rho_{\mathrm{mat}} + \rho_{\mathrm{rad}} + \rho_\Lambda)$$

物質　輻射　宇宙項（ダークエネルギー）

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

$$p_\Lambda = -\frac{\Lambda}{8\pi G}$$

とすると，

負の圧力

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_{i=\mathrm{mat,rad},\Lambda}(\rho_i + 3p_i)$$

フリードマン方程式を解く

独立な方程式は２本

$$\frac{\dot{a}^2}{a^2} + \frac{K}{a^2} = \frac{8\pi G}{3}(\rho_{\rm mat} + \rho_{\rm rad} + \rho_\Lambda)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_{i={\rm mat,rad},\Lambda} (\rho_i + 3p_i)$$

K, Λ は定数（与えられているとする）

未知なのは a, ρ, p

解くには独立な式がさらに１本必要

$\longrightarrow$ 状態方程式 $p = p(\rho)$

状態方程式

よく使われる状態方程式(一般に $\boldsymbol{p}=\boldsymbol{w}\,\rho$)

- 非相対論的物質:(冷たい)物質 $p=0$
- 相対論的物質:輻射(熱い物質)，光子 $p=\dfrac{\rho}{3}$
- 宇宙項（宇宙定数） $p=-\rho$

光子気体の状態方程式：

空洞内の電磁波は定在波

$$\nu_n=\frac{c}{2L}n \quad (n=1,2,\cdots) \longrightarrow \frac{\delta\nu_n}{\nu_n}=-\frac{\delta L}{L}=-\frac{1}{3}\frac{\delta V}{V}$$

断熱変化では，$\delta U=-p\delta V$

光子のエネルギー変化は $\delta U=h\delta\nu_n \longrightarrow -\dfrac{h\nu_n}{3}\dfrac{\delta V}{V}=-p\delta V$

よって　$p=\dfrac{1}{3}\rho$

状態方程式

ところで，

$$\frac{\dot{a}^2}{a^2} + \frac{K}{a^2} = \frac{8\pi G}{3}\rho$$

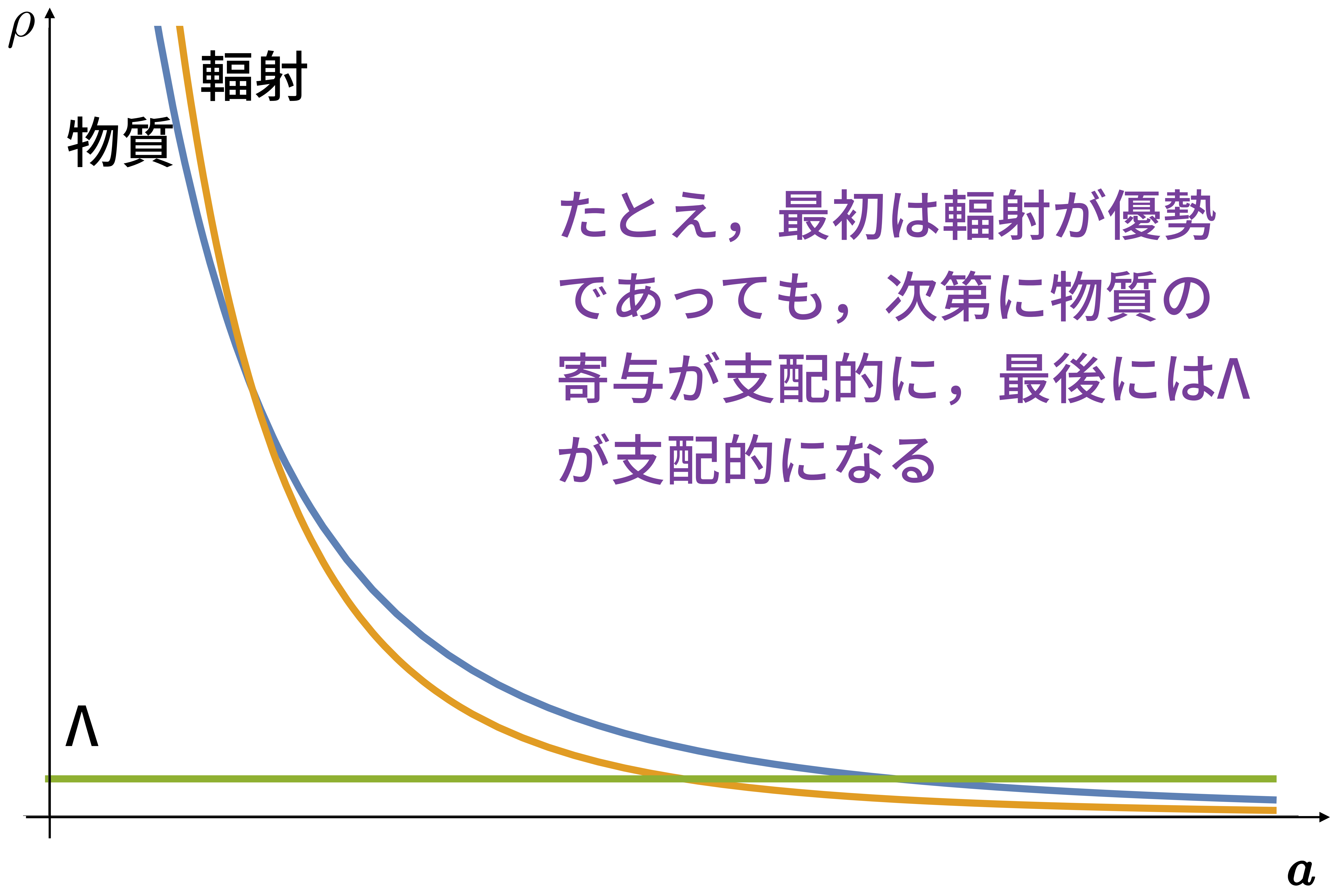
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\longrightarrow \quad \dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

エネルギー密度の進化を記述

- 物質 $p = 0$ $\quad \dfrac{\dot{\rho}}{\rho} = -3\dfrac{\dot{a}}{a} \longrightarrow \rho \propto a^{-3}$
- 輻射 $p = \dfrac{\rho}{3}$ $\quad \dfrac{\dot{\rho}}{\rho} = -4\dfrac{\dot{a}}{a} \longrightarrow \rho \propto a^{-4}$
- 宇宙項 $p = -\rho$ $\quad \dot{\rho} = 0 \longrightarrow \rho = (\text{定数})$

エネルギー密度の推移



たとえ，最初は輻射が優勢であっても，次第に物質の寄与が支配的に，最後にはΛが支配的になる

フリードマン方程式の積分

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}(\rho_{\rm mat} + \rho_{\rm rad} + \rho_\Lambda)$$

$$= H^2 \frac{\rho_{\rm mat} + \rho_{\rm rad} + \rho_\Lambda}{\rho_c}$$

現在の値を代入すると，

$$H_0^2 + \frac{K}{a_0^2} = H_0^2 \frac{\rho_{\rm mat,0} + \rho_{\rm rad,0} + \rho_{\Lambda,0}}{\rho_{c,0}}$$

これらからKを消去してまとめると，

$$\frac{H^2}{H_0^2} = \frac{\rho_{\rm mat} + \rho_{\rm rad} + \rho_\Lambda}{\rho_{c,0}} + \frac{a_0^2}{a^2}\left(1 - \frac{\rho_{\rm mat,0} + \rho_{\rm rad,0} + \rho_{\Lambda,0}}{\rho_{c,0}}\right)$$

曲率の情報

フリードマン方程式の積分

$$\frac{H^2}{H_0^2} = \frac{\rho_{\rm mat} + \rho_{\rm rad} + \rho_\Lambda}{\rho_{c,0}} + \frac{a_0^2}{a^2}\left(1 - \frac{\rho_{\rm mat,0} + \rho_{\rm rad,0} + \rho_{\Lambda,0}}{\rho_{c,0}}\right)$$

エネルギー密度の時間変化をとりいれる

- 物質 $\rho \propto a^{-3}$ $\longrightarrow$ $\rho_{\rm mat}(t) = \rho_{\rm mat,0}\dfrac{a_0^3}{a(t)^3}$
- 輻射 $\rho \propto a^{-4}$ $\longrightarrow$ $\rho_{\rm rad}(t) = \rho_{\rm rad,0}\dfrac{a_0^4}{a(t)^4}$
- 宇宙項 $\rho = (\text{定数})$ $\longrightarrow$ $\rho_\Lambda(t) = \rho_{\Lambda,0}\ (\text{一定})$

$$\frac{H^2}{H_0^2} = \frac{a_0^3}{a^3}\Omega_{\rm mat,0} + \frac{a_0^4}{a^4}\Omega_{\rm rad,0} + \Omega_{\Lambda,0} + \frac{a_0^2}{a^2}(1 - \Omega_0)$$

$$\Omega_i = \frac{\rho_i}{\rho_c} \qquad \Omega_{i,0} = \frac{\rho_{i,0}}{\rho_{c,0}} \qquad \Omega_0 = \Omega_{\rm m,0} + \Omega_{\rm m,0} + \Omega_{\Lambda,0}$$

フリードマン方程式の積分

$$\frac{H}{H_0} = \sqrt{\frac{a_0^3}{a^3}\Omega_{\mathrm{mat},0} + \frac{a_0^4}{a^4}\Omega_{\mathrm{rad},0} + \Omega_{\Lambda,0} + \frac{a_0^2}{a^2}(1-\Omega_0)}$$

$$\downarrow$$

$$\frac{1}{H_0}\frac{\dot{a}}{a_0} = \sqrt{\frac{a_0}{a}\Omega_{\mathrm{mat},0} + \frac{a_0^2}{a^2}\Omega_{\mathrm{rad},0} + \frac{a^2}{a_0^2}\Omega_{\Lambda,0} + (1-\Omega_0)}$$

$y(t) = \dfrac{a(t)}{a_0}$ のように規格化しておくと，$y(t_0) = 1$

$$\frac{1}{H_0}\frac{dy}{dt} = \sqrt{\frac{1}{y}\Omega_{\mathrm{mat},0} + \frac{1}{y^2}\Omega_{\mathrm{rad},0} + y^2\Omega_{\Lambda,0} + (1-\Omega_0)}$$

フリードマン方程式の積分

$$\frac{1}{H_0}\frac{dy}{dt} = \sqrt{\frac{1}{y}\Omega_{\mathrm{mat},0} + \frac{1}{y^2}\Omega_{\mathrm{rad},0} + y^2\Omega_{\Lambda,0} + (1-\Omega_0)}$$

これは変数分離で積分可能

$$H_0 t = \int_0^y \frac{dy}{\sqrt{y^{-1}\Omega_{\mathrm{mat},0} + y^{-2}\Omega_{\mathrm{rad},0} + y^2\Omega_{\Lambda,0} + (1-\Omega_0)}}$$

参考： $\Omega_{\mathrm{mat},0} \sim 0.3$ $\Omega_{\mathrm{rad},0} \sim 0$ $\Omega_{\Lambda,0} \sim 0.7$

単純な場合の解

$$H_0 t = \int_0^y \frac{dy}{\sqrt{y^{-1}\Omega_{\mathrm{mat},0} + y^{-2}\Omega_{\mathrm{rad},0} + y^2\Omega_{\Lambda,0} + (1-\Omega_0)}}$$

平坦な宇宙(Ω_0=1)で，かつ

物質優勢な宇宙(Ω_{mat}以外を無視する)

$$H_0 t = \int_0^y \frac{dy}{\sqrt{y^{-1}\Omega_{\mathrm{mat},0}}} = \frac{1}{\sqrt{\Omega_{\mathrm{mat},0}}}\frac{2}{3}y^{3/2} \qquad a(t) \propto t^{2/3}$$

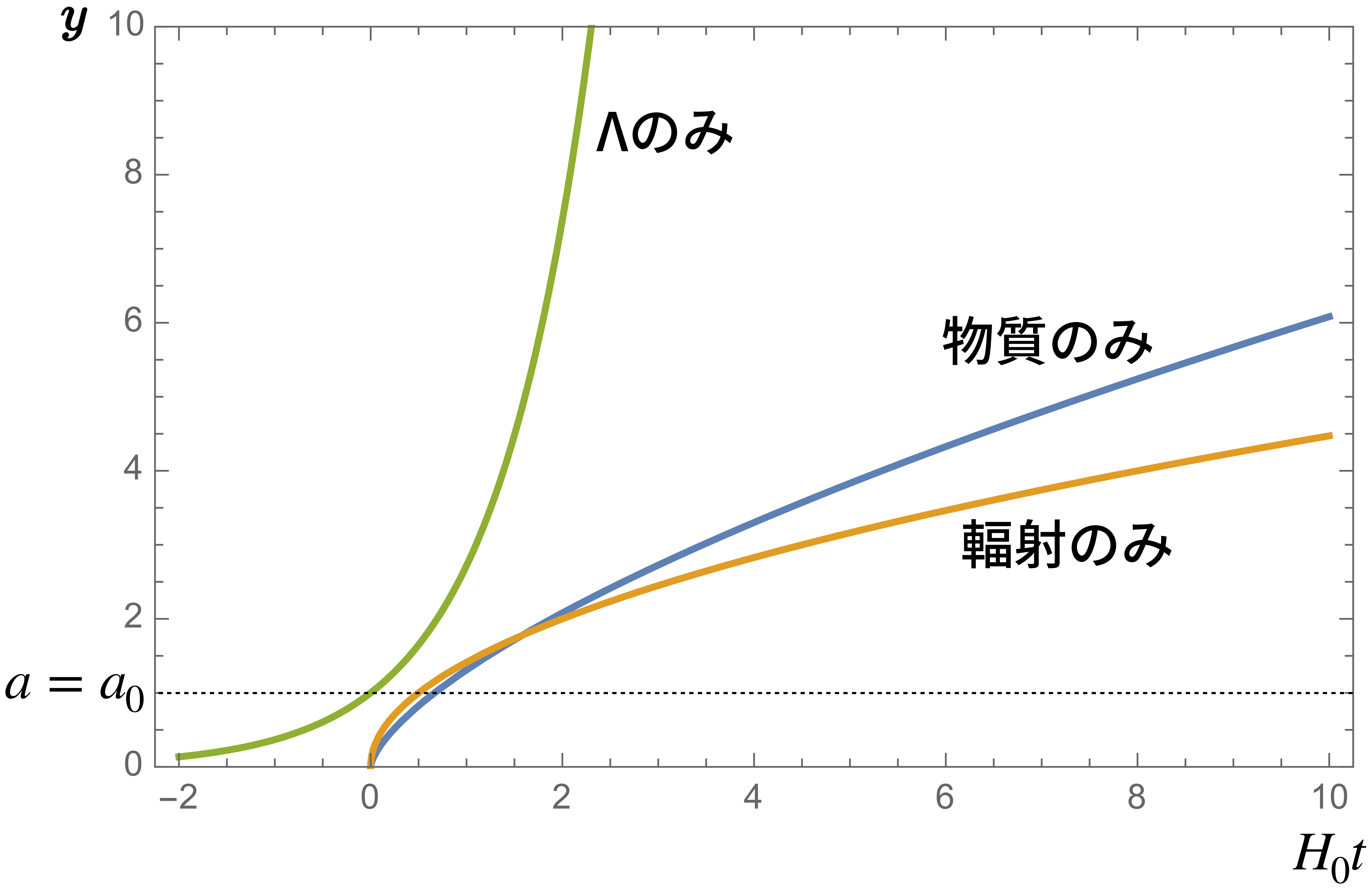
輻射優勢な宇宙(Ω_{rad}以外を無視する)

$$H_0 t = \int_0^y \frac{y\,dy}{\sqrt{\Omega_{\mathrm{rad},0}}} = \frac{1}{\sqrt{\Omega_{\mathrm{rad},0}}}\frac{1}{2}y^2 \qquad a(t) \propto t^{1/2}$$

ダークエネルギー優勢な宇宙(Ω_Λ以外を無視する)

$$H_0 t = \int_0^y \frac{dy}{y\sqrt{\Omega_{\Lambda,0}}} = \frac{1}{\sqrt{\Omega_{\Lambda,0}}}\log y \qquad a(t) = a_0 e^{\sqrt{\Omega_{\Lambda,0}}H_0 t}$$

単純な場合の解のふるまい



フリードマン解

今度は，平坦な宇宙に限らず，物質優勢の宇宙を考える

$$H_0 t = \int_0^y \frac{\sqrt{y}dy}{\sqrt{\Omega_{\mathrm{mat},0} + (1 - \Omega_{\mathrm{m},0})y}}$$

平坦な宇宙の場合： $\Omega_{\mathrm{m},0} = 1$

$$\frac{a(t)}{a_0} = \left(\frac{3}{2}H_0 t\right)^{2/3}$$

フリードマン解

今度は，平坦な宇宙に限らず，物質優勢の宇宙を考える

$$H_0 t = \int_0^y \frac{\sqrt{y}dy}{\sqrt{\Omega_{\mathrm{mat},0} + (1-\Omega_{\mathrm{m},0})y}}$$

閉じた宇宙の場合： $\Omega_{\mathrm{m},0} > 1$

$$H_0 t = \frac{\Omega_{\mathrm{mat},0}}{(\Omega_{\mathrm{mat},0}-1)^{3/2}}\left[\arcsin\sqrt{x} - \sqrt{x(1-x)}\right]$$

$$x = \frac{\Omega_{\mathrm{mat},0}-1}{\Omega_{\mathrm{mat},0}}y$$

あるいは，媒介変数を利用して

$$\frac{a(\theta)}{a_0} = \frac{\Omega_{\mathrm{mat},0}}{2(\Omega_{\mathrm{mat},0}-1)}(1-\cos\theta)$$

$$H_0 t(\theta) = \frac{\Omega_{\mathrm{mat},0}}{2(\Omega_{\mathrm{mat},0}-1)^{3/2}}(\theta - \sin\theta)$$

フリードマン解

今度は，平坦な宇宙に限らず，物質優勢の宇宙を考える

$$H_0 t = \int_0^y \frac{\sqrt{y}dy}{\sqrt{\Omega_{\mathrm{mat},0} + (1-\Omega_{\mathrm{m},0})y}}$$

開いた宇宙の場合： $\Omega_{\mathrm{mat},0} < 1$

$$H_0 t = \frac{\Omega_{\mathrm{mat},0}}{(1-\Omega_{\mathrm{mat},0})^{3/2}}\left[\sqrt{x(1+x)} - \sinh^{-1}\sqrt{x}\right]$$

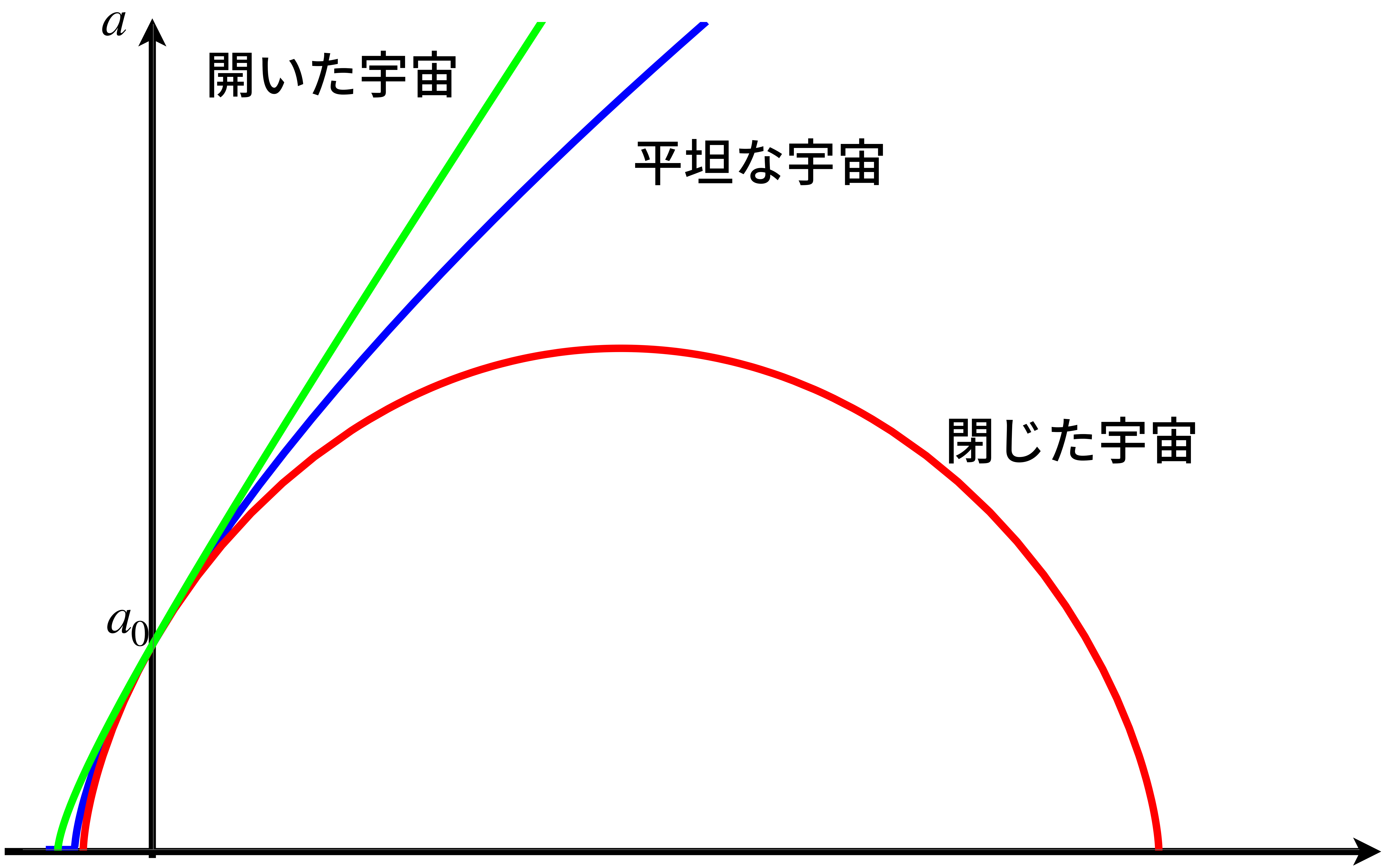
$$x = \frac{1-\Omega_{\mathrm{mat},0}}{\Omega_{\mathrm{mat},0}}y$$

あるいは，媒介変数を利用して

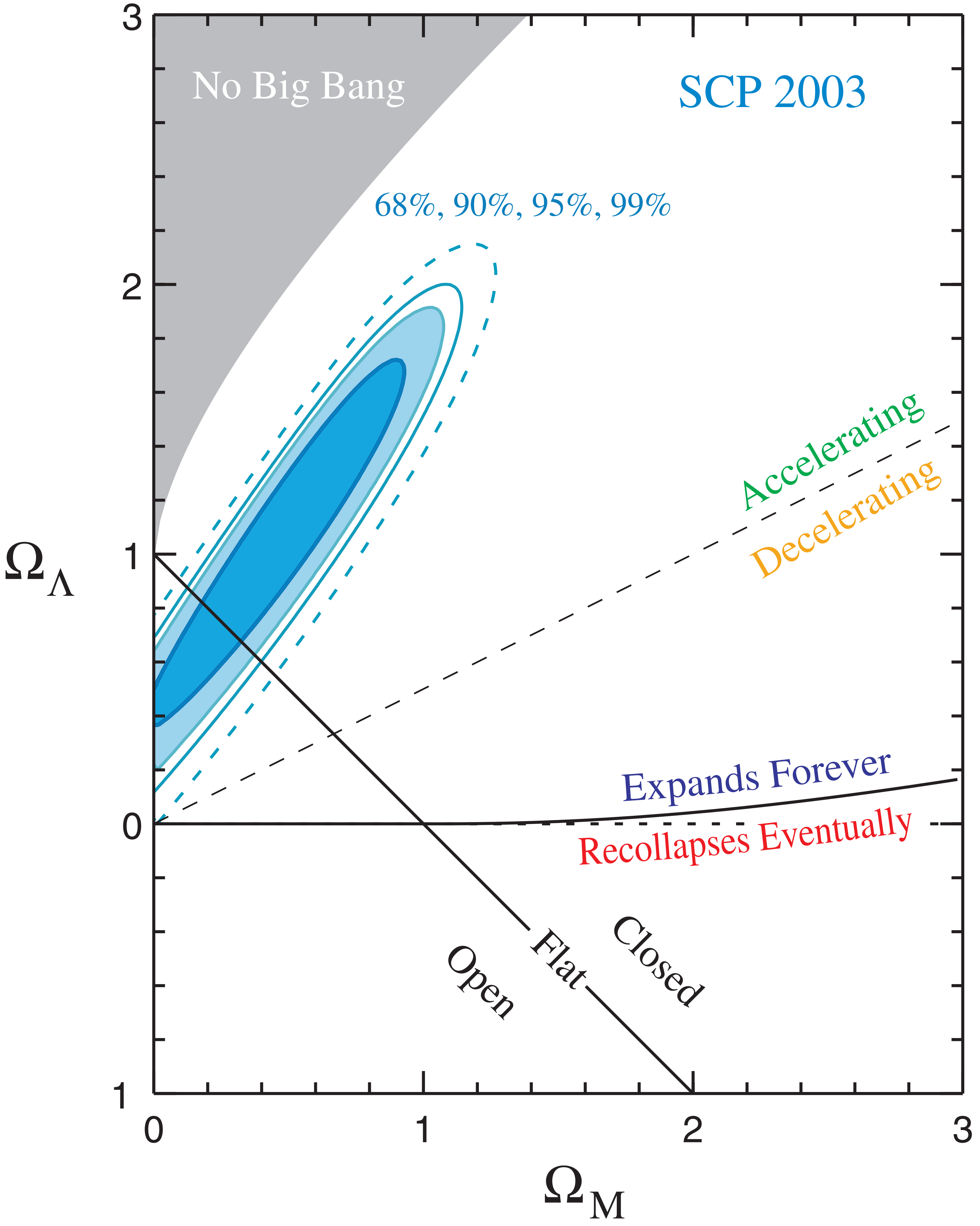
$$\frac{a(\xi)}{a_0} = \frac{\Omega_{\mathrm{mat},0}}{2(1-\Omega_{\mathrm{mat},0})}(\cosh\xi - 1)$$

$$H_0 t(\xi) = \frac{\Omega_{\mathrm{mat},0}}{2(1-\Omega_{\mathrm{mat},0})^{3/2}}(\sinh\xi - \xi)$$

フリードマン解



現在の観測値



R.A.Knop et al., Astrophysical Journal 598,102 (2003)

平坦な宇宙が示唆される

物質30%

ダークエネルギー70%

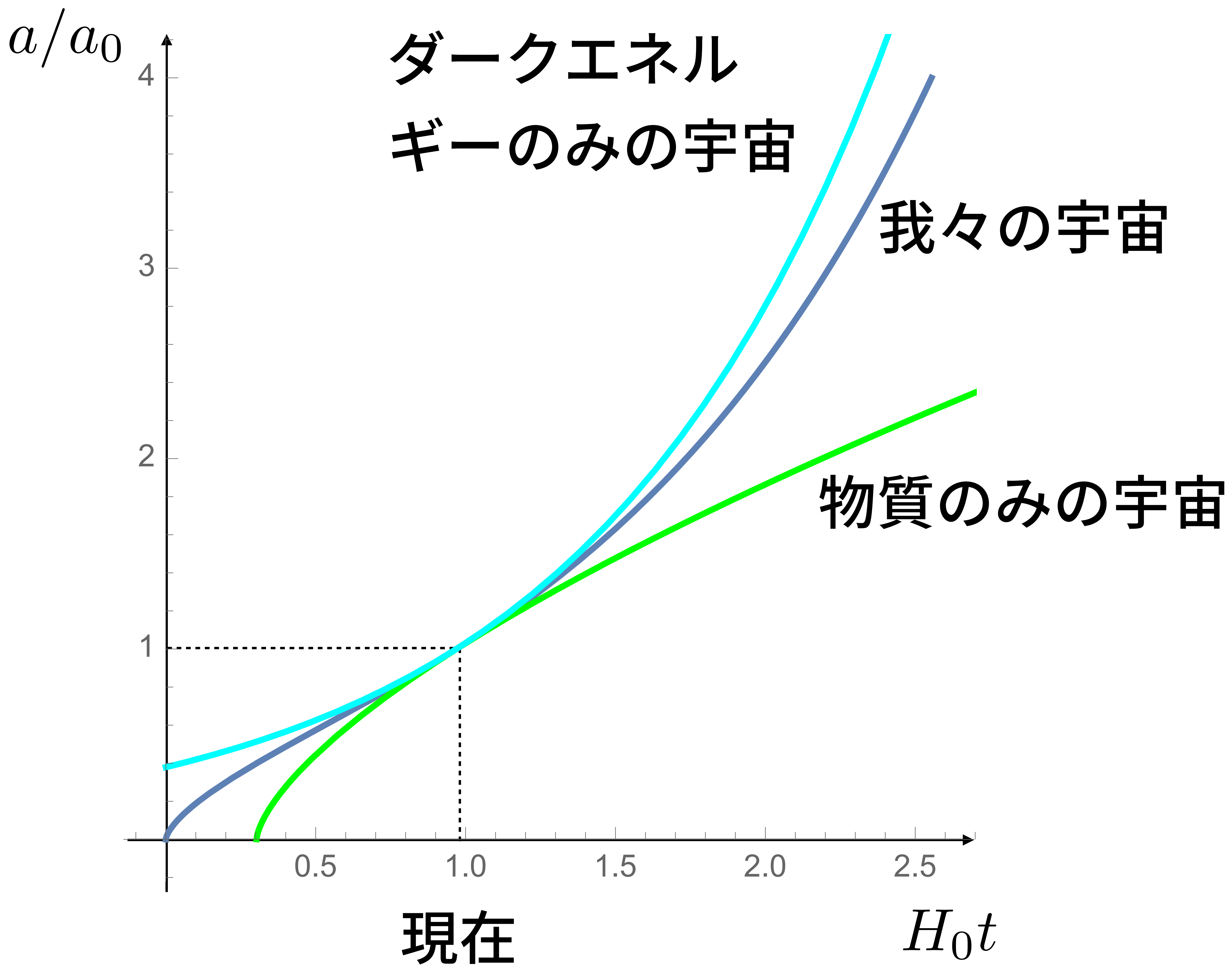
我々の宇宙の膨張

$$\Omega_{\mathrm{rad},0} = 0 \quad \Omega_{\mathrm{mat},0} + \Omega_\Lambda = 1$$ の場合

$$H_0 t = \int_0^y \frac{dy}{\sqrt{y^{-1}\Omega_{\mathrm{mat},0} + y^2\Omega_\Lambda}} = \int_0^y \frac{dy}{\sqrt{y^{-1}\Omega_{\mathrm{mat},0} + y^2(1-\Omega_{\mathrm{mat},0})}}$$

$$\omega = \left(\frac{\Omega_{\mathrm{mat},0}}{\Omega_\Lambda}\right)^{1/3} \sim 0.75$$ とすると，

$$H_0 t = \frac{2}{3\sqrt{\Omega_\Lambda}} \log\left[\left(\frac{y}{\omega}\right)^{3/2} + \sqrt{1+\left(\frac{y}{\omega}\right)^3}\right]$$


a/a_0
4
3
2
1
ダークエネルギーのみの宇宙
我々の宇宙
物質のみの宇宙
0.5
1.0
1.5
2.0
2.5
現在
H_0t