『微分積分』 第 6 章 演習問題の解答

問題 6.1

(1) $f(x) = x + 1$ として

$$x_k = \frac{ak}{n} \quad (\,0 \leqq k \leqq n\,) \quad , \quad \Delta x = \frac{a}{n}$$

とおけば

$$\begin{aligned}
\sum_{k=1}^{n} f(x_k)\Delta x &= \sum_{k=1}^{n} \left\{ \left(\frac{ak}{n} + 1 \right) \cdot \frac{a}{n} \right\} = \frac{a^2}{n^2} \sum_{k=1}^{n} k + \frac{a}{n} \sum_{k=1}^{n} 1 \\
&= \frac{a^2}{n^2} \cdot \frac{1}{2} n(\,n+1\,) + \frac{a}{n} \cdot n = \frac{a^2}{2} \cdot \frac{n+1}{n} + a \\
&= \frac{a^2}{2} \left(1 + \frac{1}{n} \right) + a \to \frac{a^2}{2} + a \quad (\,n \to \infty\,) .
\end{aligned}$$

$$\therefore \quad \int_0^a (\,x+1\,)\,dx = \frac{a^2}{2} + a \,.$$

(2) $f(x) = x^3$ として

$$x_k = \frac{k}{n} \quad (\,0 \leqq k \leqq n\,) \quad , \quad \Delta x = \frac{1}{n}$$

とおけば

$$\begin{aligned}
\sum_{k=1}^{n} f(x_k)\Delta x &= \sum_{k=1}^{n} \left\{ \left(\frac{k}{n} \right)^3 \cdot \frac{1}{n} \right\} = \frac{1}{n^4} \sum_{k=1}^{n} k^3 = \frac{1}{n^4} \cdot \left\{ \frac{1}{2} n(\,n+1\,) \right\}^2 \\
&= \frac{1}{4} \left(1 + \frac{1}{n} \right)^2 \to \frac{1}{4} \quad (\,n \to \infty\,) .
\end{aligned}$$

$$\therefore \quad \int_0^1 x^3\,dx = \frac{1}{4} \,.$$

(3) $f(x) = e^x$ として

$$x_k = \frac{k}{n} \quad (\,0 \leqq k \leqq n\,) \quad , \quad \Delta x = \frac{1}{n}$$

とおけば

$$\begin{aligned}
\sum_{k=1}^{n} f(x_k)\Delta x &= \sum_{k=1}^{n} \left(e^{\frac{k}{n}} \cdot \frac{1}{n} \right) = \frac{1}{n} \sum_{k=1}^{n} (\,e^{\frac{1}{n}}\,)^k \\
&= \frac{1}{n} \cdot \frac{e^{\frac{1}{n}} \left\{ (\,e^{\frac{1}{n}}\,)^n - 1 \right\}}{e^{\frac{1}{n}} - 1} = \frac{1}{n} \cdot \frac{e^{\frac{1}{n}} (\,e-1\,)}{e^{\frac{1}{n}} - 1} .
\end{aligned}$$

ここで $h=\dfrac{1}{n}$ とすれば，$n\to\infty$ のとき $h\to 0$ であり，

$$\sum_{k=0}^{n-1} f(x_k)\Delta x \;=\; e^h\cdot(\,e-1\,)\cdot\frac{h}{e^h-1} \;=\; e^h\cdot(\,e-1\,)\cdot\frac{1}{\dfrac{e^h-1}{h}} \;\to\; e-1 \quad (\,h\to 0\,)\,.$$

$$\therefore\quad \int_0^1 e^x\,dx = e-1\,.$$

問題 6.2

(1) $\displaystyle\int_{-1}^{2}(\,x^3-2x+3\,)\,dx = \left[\,\frac{1}{4}x^4-x^2+3x\,\right]_{-1}^{2} = (\,4-4+6\,)-\left(\frac{1}{4}-1-3\right) = \frac{39}{4}\,.$

(2) $\displaystyle\int_1^4 \frac{dx}{x} = \Big[\log x\Big]_1^4 = \log 4-\log 1 = 2\log 2\,.$

(3) $\displaystyle\int_1^4 \frac{dx}{x^2} = \int_1^4 x^{-2}\,dx = \left[\,-\frac{1}{x}\,\right]_1^4 = -\frac{1}{4}+1 = \frac{3}{4}\,.$

(4) $\displaystyle\int_1^2 \frac{dx}{x^3} = \int_1^2 x^{-3}\,dx = \left[\,\frac{1}{-2}x^{-2}\,\right]_1^2 = \left[\,-\frac{1}{2x^2}\,\right]_1^2 = -\frac{1}{8}-\left(-\frac{1}{2}\right) = \frac{3}{8}\,.$

(5) $\displaystyle\int_1^4 \sqrt{x}\,dx = \int_1^4 x^{\frac{1}{2}}\,dx = \left[\,\frac{2}{3}x^{\frac{3}{2}}\,\right]_1^4 = \frac{16}{3}-\frac{2}{3} = \frac{14}{3}\,.$

(6) $\displaystyle\int_1^8 \sqrt[3]{x}\,dx = \int_1^8 x^{\frac{1}{3}}\,dx = \left[\,\frac{1}{\frac{1}{3}+1}x^{\frac{1}{3}+1}\,\right]_1^8 = \left[\,\frac{3}{4}x^{\frac{4}{3}}\,\right]_1^8 = \frac{3}{4}(\,16-1\,) = \frac{45}{4}\,.$

(7) $\displaystyle\int_1^4 \frac{dx}{x\sqrt{x}} = \int_1^4 x^{-\frac{3}{2}}\,dx = \Big[\,-2x^{-\frac{1}{2}}\,\Big]_1^4 = \left[\,-\frac{2}{\sqrt{x}}\,\right]_1^4 = -1-(\,-2\,) = 1\,.$

(8) $\displaystyle\int_1^4 \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx = \int_1^4 \left(x-2+\frac{1}{x}\right)dx = \left[\,\frac{1}{2}x^2-2x+\log|\,x\,|\,\right]_1^4$

$\displaystyle= (\,8-8+\log 4\,)-\left(\frac{1}{2}-2+\log 1\right) = \frac{3}{2}+2\log 2\,.$

(9) $\displaystyle\int_{-1}^{2} e^x\,dx = \Big[\,e^x\,\Big]_{-1}^{2} = e^2-e^{-1} = e^2-\frac{1}{e}\,.$

(10) $\displaystyle\int_{\log 2}^{\log 3} e^x\,dx = \Big[\,e^x\,\Big]_{\log 2}^{\log 3} = e^{\log 3}-e^{\log 2} = 3-2 = 1\,.$

(11) $\displaystyle \int_0^3 2^x\,dx = \left[\,\frac{2^x}{\log 2}\,\right]_0^3 = \frac{2^3}{\log 2} - \frac{2^0}{\log 2} = \frac{8}{\log 2} - \frac{1}{\log 2} = \frac{7}{\log 2}$.

(12) $\displaystyle \int_0^{\log_3 2} 3^x\,dx = \left[\,\frac{3^x}{\log 3}\,\right]_0^{\log_3 2} = \frac{3^{\log_3 2}}{\log 3} - \frac{3^0}{\log 3} = \frac{2}{\log 3} - \frac{1}{\log 3} = \frac{1}{\log 3}$.

(13) $\displaystyle \int_0^{\frac{\pi}{2}} \sin x\,dx = \Big[-\cos x\Big]_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2} + \cos 0 = 1$.

(14) $\displaystyle \int_0^{\frac{\pi}{3}} \frac{dx}{\cos^2 x} = \Big[\tan x\Big]_0^{\frac{\pi}{3}} = \tan\frac{\pi}{3} - \tan 0 = \sqrt{3}$.

(15) $\displaystyle \int_0^{\frac{2\pi}{3}} (\,2\sin x - \cos x\,)\,dx = \Big[\,-2\cos x - \sin x\,\Big]_0^{\frac{2\pi}{3}}$

$\displaystyle = \left(-2\cos\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right) - (\,-2\cos 0 - \sin 0\,) = \left(1 - \frac{\sqrt{3}}{2}\right) - (\,-2 - 0\,)$

$\displaystyle = 3 - \frac{\sqrt{3}}{2}$.

(16) $\displaystyle \int_0^{\frac{\pi}{4}} \tan^2 x\,dx = \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos^2 x} - 1\right) dx = \Big[\,\tan x - x\,\Big]_0^{\frac{\pi}{4}} = 1 - \frac{\pi}{4}$.

(17) $\displaystyle \int_1^{\sqrt{3}} \frac{dx}{x^2+1} = \Big[\tan^{-1} x\Big]_1^{\sqrt{3}} = \tan^{-1}\sqrt{3} - \tan^{-1} 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$.

(18) $\displaystyle \int_0^{\frac{1}{\sqrt{2}}} \frac{dx}{\sqrt{1-x^2}} = \Big[\sin^{-1} x\Big]_0^{\frac{1}{\sqrt{2}}} = \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$.

問題 6.3

(1) $\displaystyle \int_0^1 (\,2x+1\,)^3 dx = \frac{1}{2}\int_0^1 (\,2x+1\,)^3 \cdot (\,2x+1\,)'\,dx = \frac{1}{2}\left[\,\frac{1}{4}(\,2x+1\,)^4\,\right]_0^1$

$\displaystyle = \frac{1}{2}\left(\frac{81}{4} - \frac{1}{4}\right) = 10$.

(2) $\displaystyle \int_0^4 \frac{dx}{3x+4} = \frac{1}{3}\int_0^4 \frac{1}{3x+4} \cdot (\,3x+4\,)'\,dx = \frac{1}{3}\Big[\log|\,3x+4\,|\Big]_0^4 = \frac{1}{3}\big(\log 16 - \log 4\big)$

$\displaystyle = \frac{2}{3}\log 2$.

(3) $\displaystyle\int_0^1 e^{2x}\,dx = \frac{1}{2}\int_0^1 e^{2x}\cdot(2x)'\,dx = \frac{1}{2}\Big[e^{2x}\Big]_0^1 = \frac{1}{2}(e^2-1)\,.$

(4) $\displaystyle\int_0^1 \sqrt{3x+1}\,dx = \int_0^1 (3x+1)^{\frac{1}{2}}\,dx = \frac{1}{3}\int_0^1 (3x+1)^{\frac{1}{2}}\cdot(3x+1)'\,dx$

$\displaystyle = \frac{1}{3}\left[\frac{2}{3}(3x+1)^{\frac{3}{2}}\right]_0^1 = \frac{1}{3}\left(\frac{16}{3}-\frac{2}{3}\right) = \frac{14}{9}\,.$

(5) $\displaystyle\int_{-2}^0 \sqrt{1-4x}\,dx = \int_{-2}^0 (1-4x)^{\frac{1}{2}}\,dx = -\frac{1}{4}\int_{-2}^0 (1-4x)^{\frac{1}{2}}\cdot(1-4x)'dx$

$\displaystyle = -\frac{1}{4}\left[\frac{2}{3}(1-4x)^{\frac{3}{2}}\right]_{-2}^0 = -\frac{1}{6}(1-27) = \frac{13}{3}\,.$

(6) $\displaystyle\int_2^5 \frac{dx}{\sqrt{6-x}} = \int_2^5 (6-x)^{-\frac{1}{2}}\,dx = -\int_2^5 (6-x)^{-\frac{1}{2}}\cdot(6-x)'dx$

$\displaystyle = -\left[2(6-x)^{\frac{1}{2}}\right]_2^5 = -(2-4) = 2\,.$

(7) $\displaystyle\int_0^{\frac{\pi}{3}} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{3}} = -\frac{1}{2}\cos\frac{2\pi}{3} - \left(-\frac{1}{2}\cos 0\right) = \frac{1}{4}+\frac{1}{2} = \frac{3}{4}\,.$

(8) $\displaystyle\int_0^1 \cos\frac{\pi x}{2}\,dx = \left[\frac{2}{\pi}\sin\frac{\pi x}{2}\right]_0^1 = \frac{2}{\pi}\sin\frac{\pi}{2} - \frac{2}{\pi}\sin 0 = \frac{2}{\pi}\,.$

(9) $\displaystyle\int_0^{\frac{\pi}{6}} \sin x\,\cos x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{6}} \sin 2x\,dx = \frac{1}{2}\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}} = -\frac{1}{4}\left(\cos\frac{\pi}{3} - \cos 0\right)$

$\displaystyle = -\frac{1}{4}\left(\frac{1}{2}-1\right) = \frac{1}{8}\,.$

(10) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x\,dx = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx = \left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$

$\displaystyle = \left(\frac{\pi}{4}-0\right) - (0-0) = \frac{\pi}{4}\,.$

(11) $\displaystyle\int_0^{\frac{2\pi}{3}} \cos^2 x\,dx = \int_0^{\frac{2\pi}{3}} \frac{1+\cos 2x}{2}\,dx = \frac{1}{2}\int_0^{\frac{2\pi}{3}} (1+\cos 2x)\,dx$

$\displaystyle = \frac{1}{2}\left[x+\frac{1}{2}\sin 2x\right]_0^{\frac{2\pi}{3}} = \frac{1}{2}\left(\frac{2\pi}{3}+\frac{1}{2}\sin\frac{4\pi}{3}\right) = \frac{\pi}{3} - \frac{\sqrt{3}}{8}\,.$

(12) $\displaystyle\int_0^{\frac{\pi}{3}} \sin^2 x \cos^2 x\,dx = \int_0^{\frac{\pi}{3}} \left(\frac{1}{2}\sin 2x\right)^2 dx = \frac{1}{4}\int_0^{\frac{\pi}{3}} \sin^2 2x\,dx = \frac{1}{4}\int_0^{\frac{\pi}{3}} \frac{1-\cos 4x}{2}\,dx$

$$= \frac{1}{8}\int_0^{\frac{\pi}{3}} (1-\cos 4x)\,dx = \frac{1}{8}\left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{3}} = \frac{1}{8}\left(\frac{\pi}{3} - \frac{1}{4}\sin\frac{4\pi}{3}\right) = \frac{\pi}{24} + \frac{\sqrt{3}}{64}\ .$$

(13) まず $\cos^4 x$ の次数を下げる.

$$\begin{aligned}\cos^4 x &= (\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2 = \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x) \\ &= \frac{1}{4}\left(1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right) = \frac{1}{8}(3 + 4\cos 2x + \cos 4x)\end{aligned}$$

であることより,

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \cos^4 x\,dx &= \frac{1}{8}\int_0^{\frac{\pi}{2}} (3 + 4\cos 2x + \cos 4x)\,dx \\ &= \frac{1}{8}\left[3x + 2\sin 2x + \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{2}} = \frac{1}{8}\cdot\frac{3\pi}{2} = \frac{3\pi}{16}\ .\end{aligned}$$

(14) $\displaystyle\int_0^{\log 2} (e^x + e^{-x})^2\,dx = \int_0^{\log 2} (e^{2x} + 2 + e^{-2x})\,dx = \left[\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x}\right]_0^{\log 2}$

$$= \left(2 + 2\log 2 - \frac{1}{4}\right) - \left(\frac{1}{2} + 0 - \frac{1}{2}\right) = \frac{7}{4} + 2\log 2\ .$$

(15) $\displaystyle\frac{1}{x^2-2x-3} = \frac{1}{(x+1)(x-3)} = \frac{a}{x+1} + \frac{b}{x-3}$ とおけば,

$$\frac{a}{x+1} + \frac{b}{x-3} = \frac{(a+b)x + (-3a+b)}{(x+1)(x-3)}$$

であるから,

$$a+b=0\ ,\quad -3a+b=1 \qquad \therefore\quad a = -\frac{1}{4}\ ,\ b = \frac{1}{4}\ .$$

従って,

$$\begin{aligned}\int_1^2 \frac{dx}{x^2-2x-3} &= \frac{1}{4}\int_1^2 \left(-\frac{1}{x+1} + \frac{1}{x-3}\right) dx \\ &= \frac{1}{4}\Big[-\log|x+1| + \log|x-3|\Big]_1^2 = \frac{1}{4}\left[\log\left|\frac{x-3}{x+1}\right|\right]_1^2 \\ &= \frac{1}{4}\left(\log\frac{1}{3} - \log 1\right) = -\frac{1}{4}\log 3\ .\end{aligned}$$

問題 6.4

(1) $$\int_0^1 e^x(e^x-1)^5\,dx=\int_0^1(e^x-1)^5\cdot(e^x-1)'\,dx=\left[\frac{1}{6}(e^x-1)^6\right]_0^1=\frac{1}{6}(e-1)^6\,.$$

(2) $$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\sin^3x\cos x\,dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\sin^3x\cdot(\sin x)'\,dx=\left[\frac{1}{4}\sin^4x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$=\frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)^4-\frac{1}{4}\left(\frac{1}{\sqrt{2}}\right)^4=\frac{9}{64}-\frac{1}{16}=\frac{5}{64}\,.$$

(3) $$\int_0^{\frac{\pi}{4}}\sin^3x\,dx=\int_0^{\frac{\pi}{4}}(1-\cos^2x)\,\sin x\,dx=-\int_0^{\frac{\pi}{4}}(1-\cos^2x)\cdot(\cos x)'\,dx$$

$$=-\left[\cos x-\frac{1}{3}\cos^3x\right]_0^{\frac{\pi}{4}}=-\left\{\left(\cos\frac{\pi}{4}-\frac{1}{3}\cos^3\frac{\pi}{4}\right)-\left(\cos 0-\frac{1}{3}\cos^3 0\right)\right\}$$

$$=-\left(\frac{1}{\sqrt{2}}-\frac{1}{6\sqrt{2}}\right)+\left(1-\frac{1}{3}\right)=-\frac{5}{6\sqrt{2}}+\frac{2}{3}=\frac{8-5\sqrt{2}}{12}\,.$$

(4) $$\int_0^{\frac{\pi}{2}}\sin^3x\,\cos^3x\,dx=\int_0^{\frac{\pi}{2}}\sin^3x\cdot\cos^2x\cdot\cos x\,dx=\int_0^{\frac{\pi}{2}}\sin^3x(1-\sin^2x)\cdot(\sin x)'\,dx$$

$$=\int_0^{\frac{\pi}{2}}(\sin^3x-\sin^5x)\cdot(\sin x)'\,dx=\left[\frac{1}{4}\sin^4x-\frac{1}{6}\sin^6x\right]_0^{\frac{\pi}{2}}=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}\,.$$

(5) $$\int_{-1}^{\sqrt{3}}\frac{x}{1+x^2}\,dx=\frac{1}{2}\int_{-1}^{\sqrt{3}}\frac{(1+x^2)'}{1+x^2}\,dx=\frac{1}{2}\left[\log(1+x^2)\right]_{-1}^{\sqrt{3}}$$

$$=\frac{1}{2}(\log 4-\log 2)=\frac{1}{2}\log 2\,.$$

(6) $$\int_1^2\frac{2x-1}{x^2-x+1}\,dx=\int_1^2\frac{(x^2-x+1)'}{x^2-x+1}\,dx=\left[\log(x^2-x+1)\right]_1^2=\log 3-\log 1=\log 3\,.$$

(7) $$\int_0^{\frac{1}{2}}\frac{x}{\sqrt{1-x^2}}\,dx=\int_0^{\frac{1}{2}}x(1-x^2)^{-\frac{1}{2}}\,dx=-\frac{1}{2}\int_0^{\frac{1}{2}}(1-x^2)^{-\frac{1}{2}}(1-x^2)'\,dx$$

$$=-\frac{1}{2}\left[2(1-x^2)^{\frac{1}{2}}\right]_0^{\frac{1}{2}}=-\frac{1}{2}\left[2\sqrt{1-x^2}\right]_0^{\frac{1}{2}}=-\frac{1}{2}(\sqrt{3}-2)=\frac{2-\sqrt{3}}{2}\,.$$

(8) $$\int_0^1\frac{x}{x^4+1}\,dx=\frac{1}{2}\int_0^1\frac{2x}{(x^2)^2+1}\,dx=\frac{1}{2}\int_0^1\frac{1}{(x^2)^2+1}\cdot(x^2)'\,dx=\frac{1}{2}\left[\tan^{-1}x^2\right]_0^1$$

$$=\frac{1}{2}\left(\frac{\pi}{4}-0\right)=\frac{\pi}{8}\,.$$

問題 6.5

(1) $$\int_1^3 \frac{dx}{x^2+3} = \left[\ \frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\ \right]_1^3 = \frac{1}{\sqrt{3}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{6\sqrt{3}} = \frac{\sqrt{3}\,\pi}{18}\ .$$

(2) $$\int_0^{2\sqrt{3}} \frac{dx}{x^2+4} = \left[\ \frac{1}{2}\tan^{-1}\frac{x}{2}\ \right]_0^{2\sqrt{3}} = \frac{1}{2}\tan^{-1}\sqrt{3} - \frac{1}{2}\tan^{-1}0 = \frac{\pi}{6}\ .$$

(3) $$\int_0^1 \frac{dx}{\sqrt{2-x^2}} = \left[\sin^{-1}\frac{x}{\sqrt{2}}\ \right]_0^1 = \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}0 = \frac{\pi}{4}\ .$$

(4) $$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{6-x^2}} = \left[\ \sin^{-1}\frac{x}{\sqrt{6}}\ \right]_0^{\sqrt{3}} = \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}0 = \frac{\pi}{4}\ .$$

(5) $$\int_0^2 \frac{dx}{x^2-2x+4} = \int_0^2 \frac{dx}{(\,x-1\,)^2+3} = \left[\ \frac{1}{\sqrt{3}}\tan^{-1}\frac{x-1}{\sqrt{3}}\ \right]_0^2$$

$$= \frac{1}{\sqrt{3}}\left\{\tan^{-1}\frac{1}{\sqrt{3}} - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right\} = \frac{1}{\sqrt{3}}\left(\ \frac{\pi}{6} + \frac{\pi}{6}\ \right) = \frac{\pi}{3\sqrt{3}} = \frac{\sqrt{3}\,\pi}{9}\ .$$

(6) $$\int_0^1 \frac{dx}{x^2+x+1} = \int_0^1 \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} = \left[\ \frac{2}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}\ \right]_0^1$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\sqrt{3} - \frac{2}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}\left(\ \frac{\pi}{3} - \frac{\pi}{6}\ \right) = \frac{\pi}{3\sqrt{3}}\ .$$

(7) $$\int_{-2}^{\sqrt{3}-1} \frac{dx}{\sqrt{3-2x-x^2}} = \int_{-2}^{\sqrt{3}-1} \frac{dx}{\sqrt{4-(\,x+1\,)^2}} = \left[\sin^{-1}\frac{x+1}{2}\ \right]_{-2}^{\sqrt{3}-1}$$

$$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{2}\ .$$

(8) $$\int_0^{1+\sqrt{2}} \frac{dx}{\sqrt{3+2x-x^2}} = \int_0^{1+\sqrt{2}} \frac{dx}{\sqrt{4-(\,x-1\,)^2}} = \left[\ \sin^{-1}\frac{x-1}{2}\ \right]_0^{1+\sqrt{2}}$$

$$= \sin^{-1}\frac{\sqrt{2}}{2} - \sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{4} - \left(-\frac{\pi}{6}\right) = \frac{5\pi}{12}\ .$$