

# Electroweak Correction for the Study of Higgs Potential in LC

*LAPTH-Minamitateya Collaboration*

LCWS05

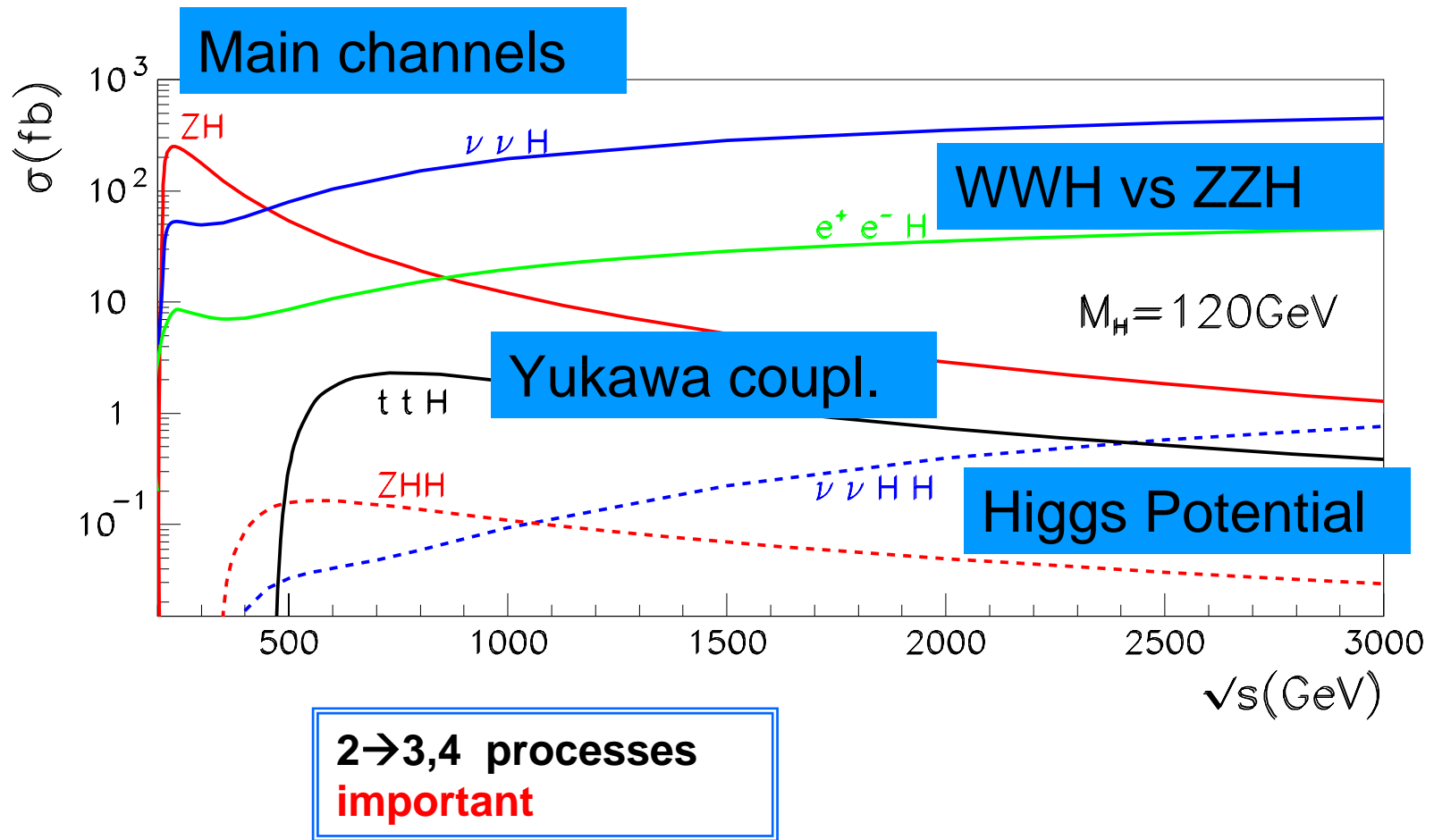
2005.3.19, Stanford U.

presented by K.Kato(Kogakuin U.)

# Introduction

- **LC** : high-precision experiments  
→ Requires the same level theoretical prediction = Needs higher order calculation
- **Higgs study** : Central target  
→ Interesting channels = multi-body final states
- **1-loop correction to  $e^+e^- \rightarrow 3, 4$ -bodies** :  
Great Progress since Sept. 2002

# Higgs@LC:tree cross sections



# number of diagrams (GRACE:NLG model)

A.Denner, J.Kublbeck,  
R.Mertig, M.Bohm,  
Z.Phys.C56(1992) 261;  
B.A.Kniehl,  
Z.Phys.C55(1992) 605.

$e^+ e^- \rightarrow$	tree	1-loop
$ZH$	4(1)	341(119)

10 years  
required to  
develop 2→3  
tools

Automated  
Systems

with(without) e-scalar couplings

*Full 1-loop RC available*

$$e^+ e^- \rightarrow \nu \bar{\nu} H$$

GRACE, PLB559(2003)252  
Denner et al., NPB660(2003)289

$$e^+ e^- \rightarrow t \bar{t} H$$

GRACE, PLB571(2003)163  
You et al., PLB571(2003)85  
Denner et al., PLB575(2003)290

$$e^+ e^- \rightarrow Z H H$$

GRACE, PLB576(2003)152  
Zhang et al., PLB578(2004)349

$$e^+ e^- \rightarrow e^+ e^- H$$

GRACE, PLB600(2004)65

$$e^+ e^- \rightarrow \nu \bar{\nu} \gamma \quad \nu = \nu_\mu, \nu_e$$

GRACE, NIM A534(2004)334

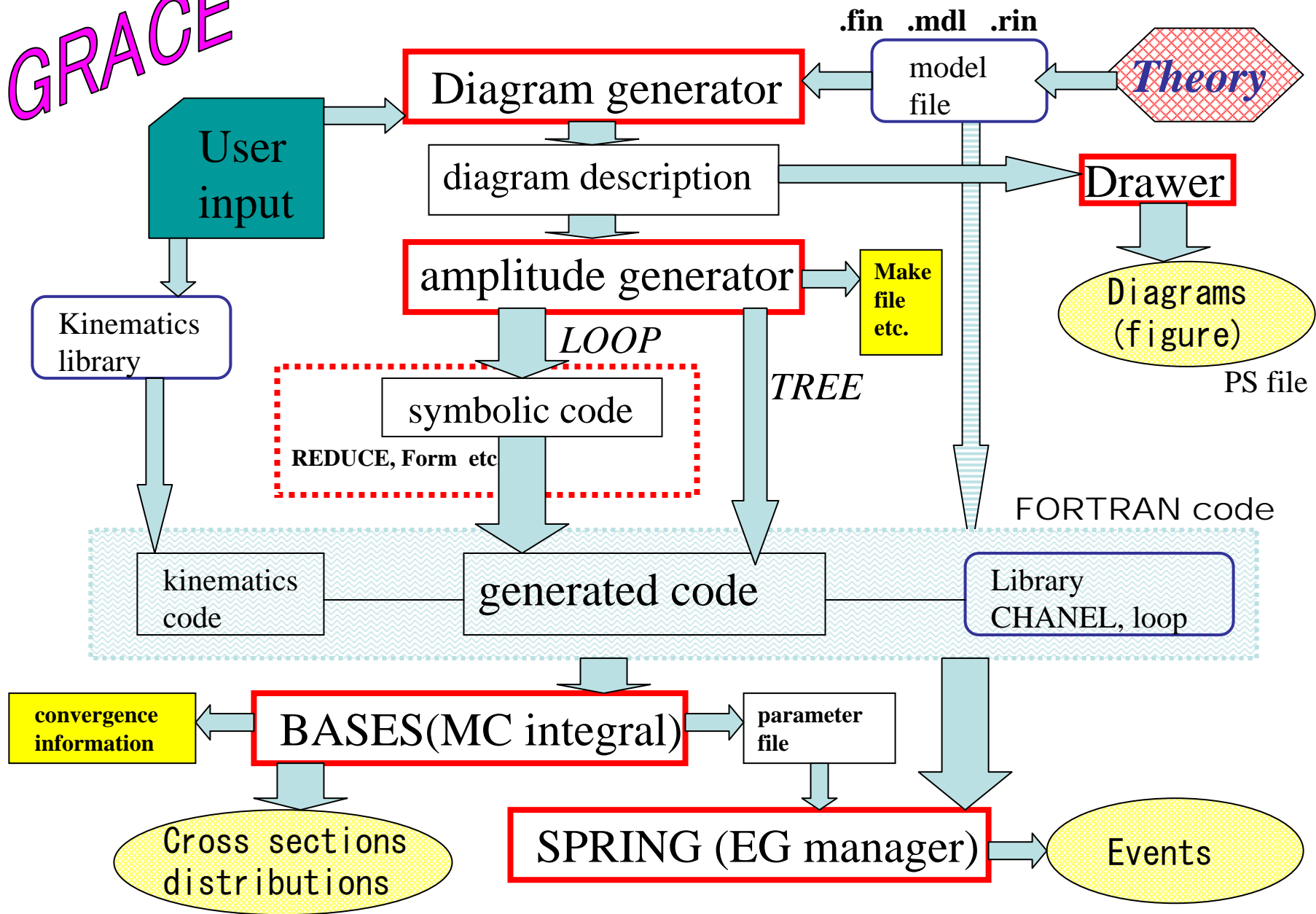
$$e^+ e^- \rightarrow \nu \bar{\nu} H H$$

GRACE, Talk by Y.Yasui at  
Durham(Sep.2004)

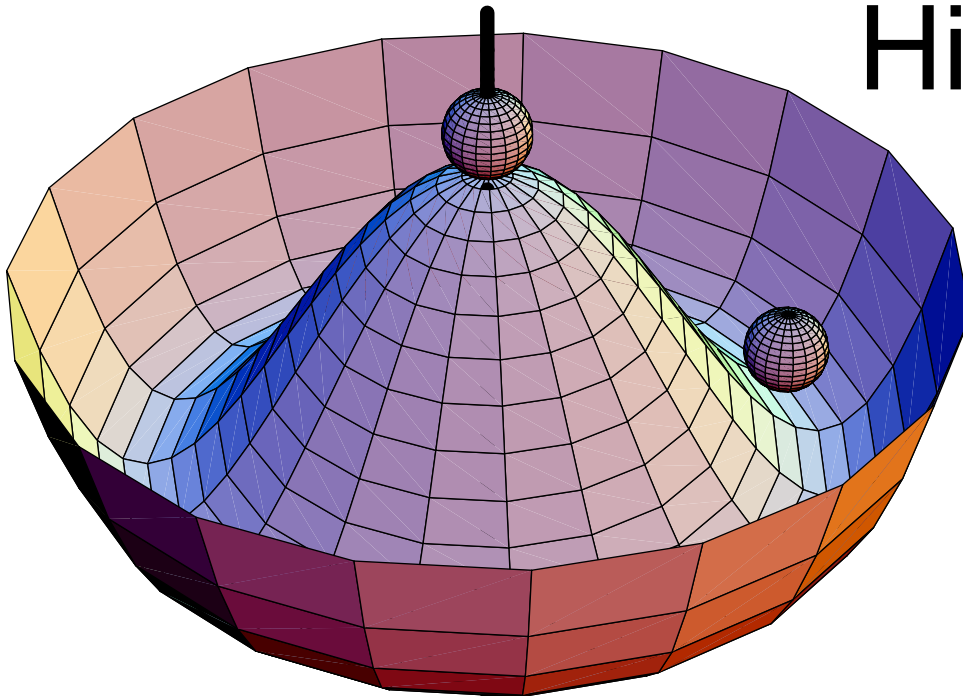
$$e^+ e^- \rightarrow \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu, \quad \bar{u} \bar{d} \bar{s} \bar{c}$$

Denner et al., hep-  
ph/0502063

# GRACE

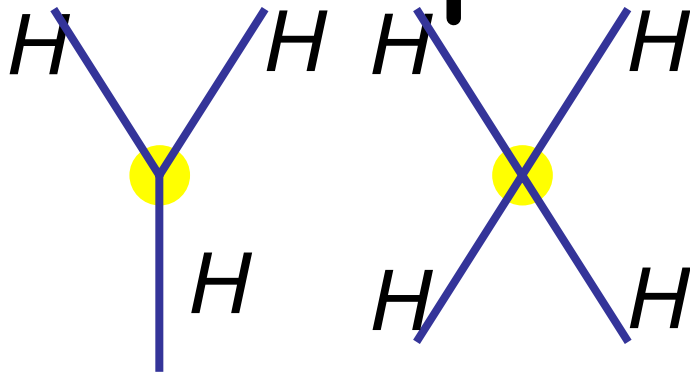


# Higgs Potential



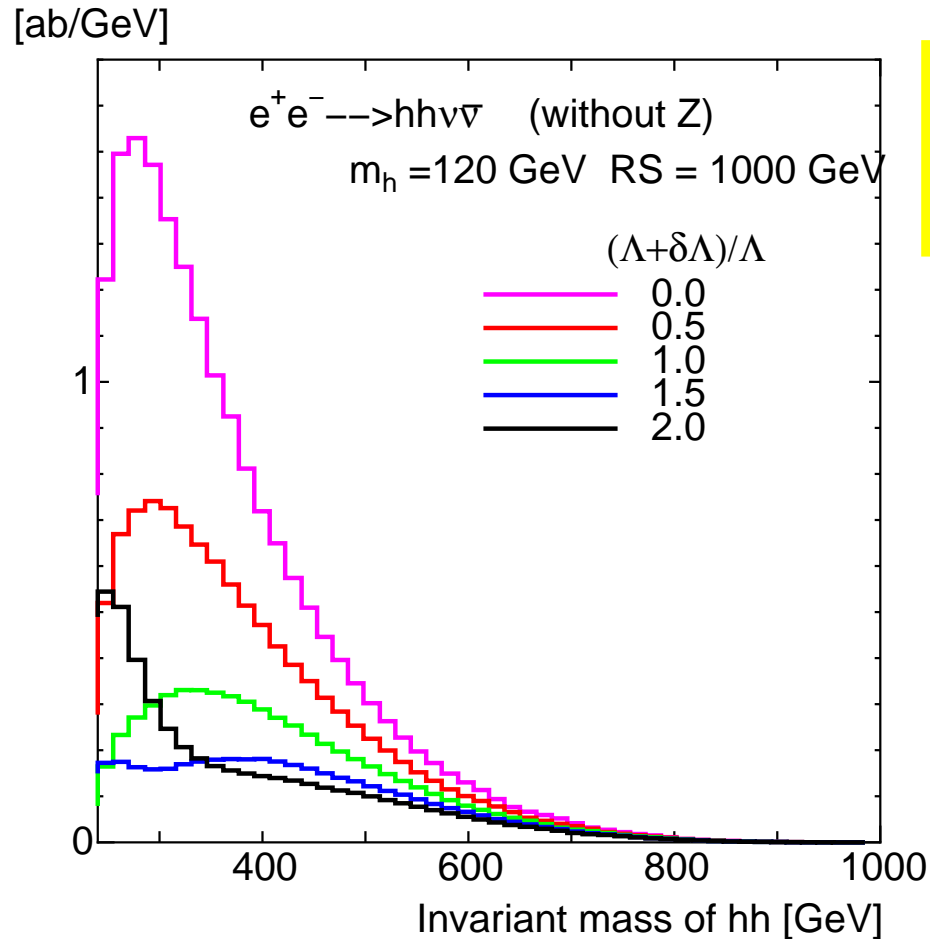
*Something required to  
provide mass  
Key for the structure of  
standard model*

*Window to New  
Physics*



Double (triple) Higgs  
production Process

$$e^+e^- \rightarrow \nu \nu HH$$



HHH coupling  
dependence  
SM:  $\delta \Lambda = 0$

tree  
old parameter set

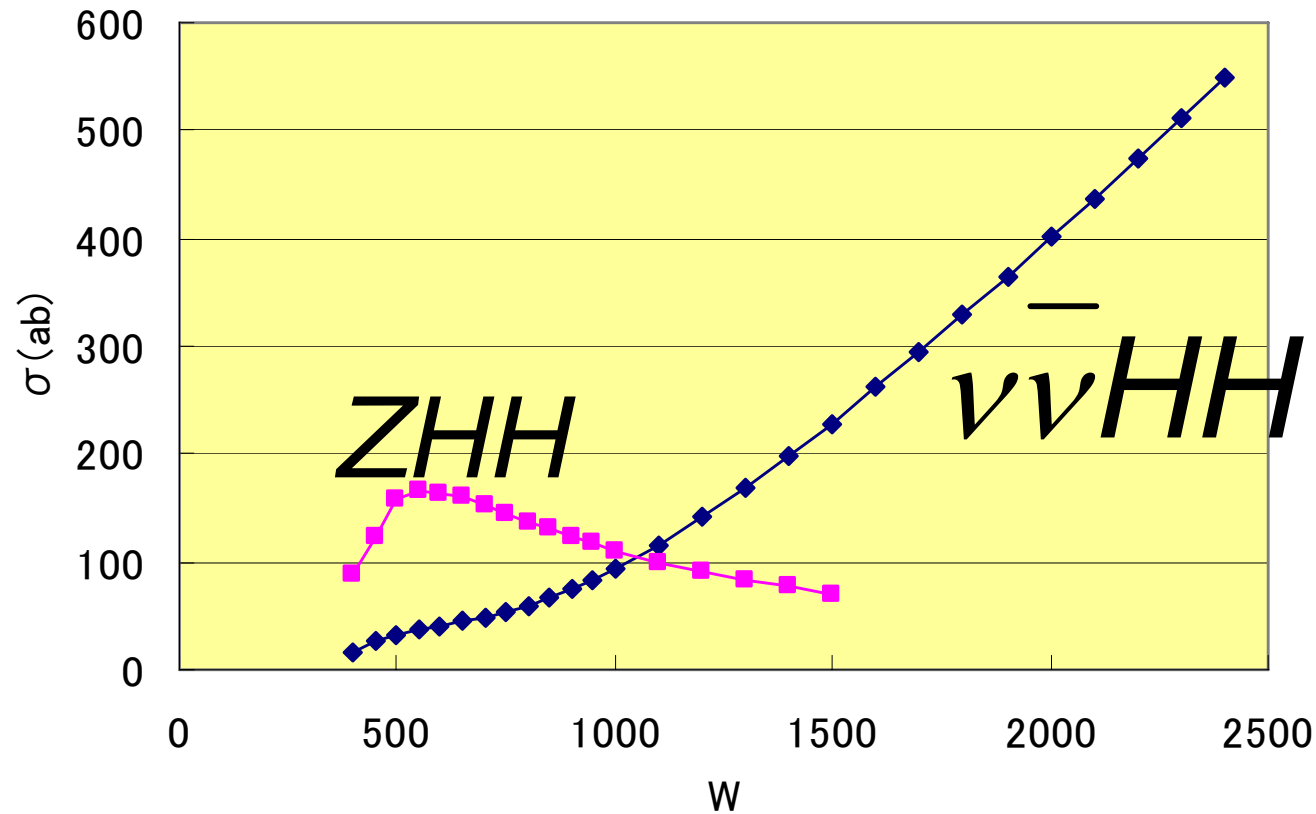


# $\nu\nu HH$ vs $ZHH$

$M_H = 120 \text{ GeV}$

ZHH/nunuHH

Tree



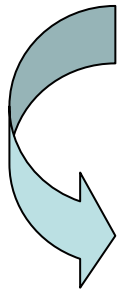
$$e^+ e^- \rightarrow ZHH$$

Low  $W$

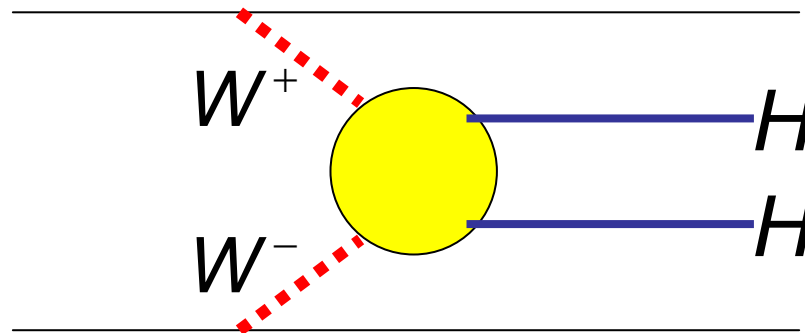
FULL calculation

$$e^+ e^- \rightarrow \nu\bar{\nu}HH$$

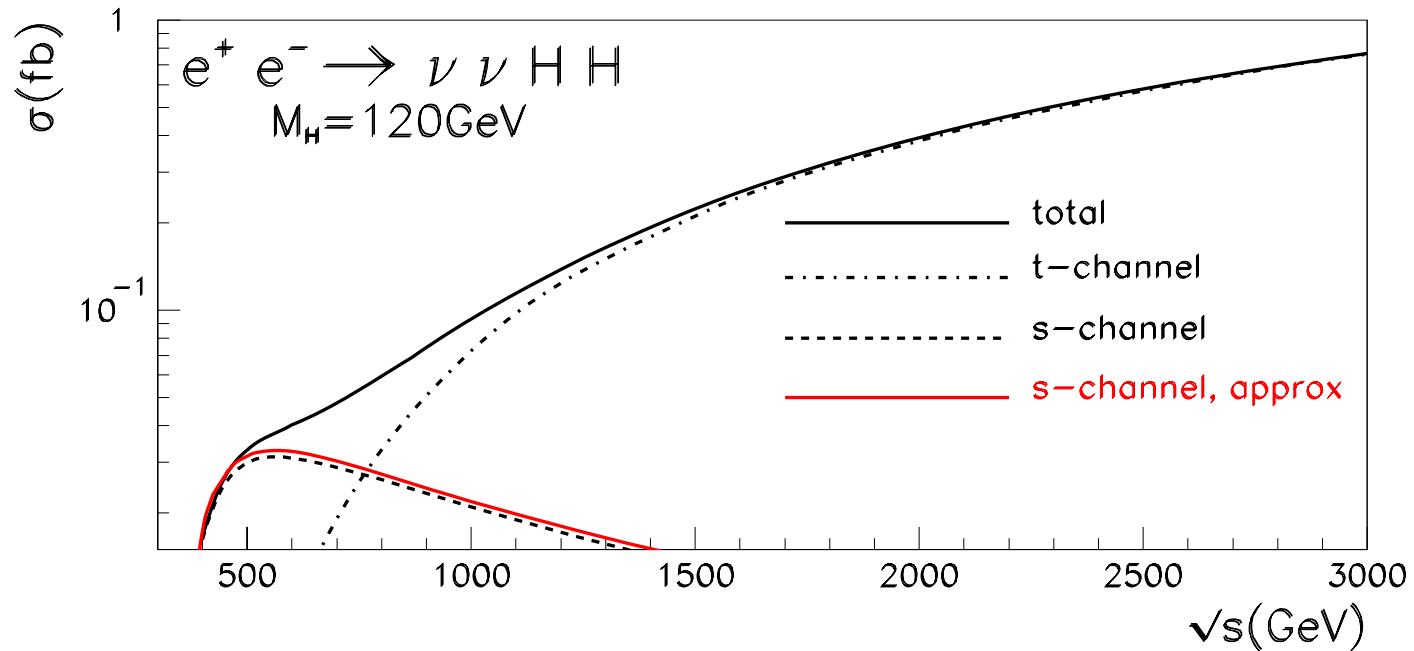
High  $W$



dominant mechanism (approximation)

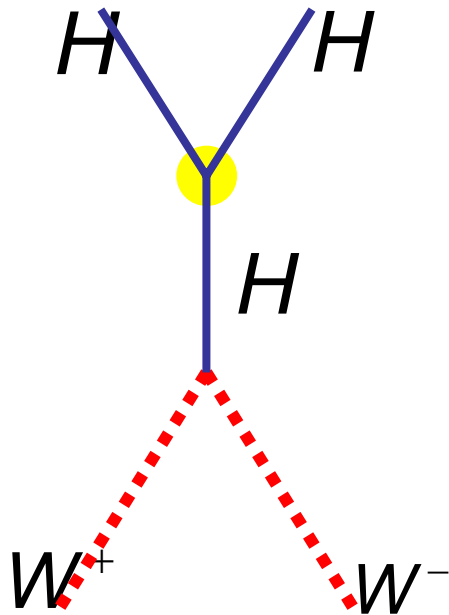


$$e^+e^- \rightarrow \nu \nu HH$$

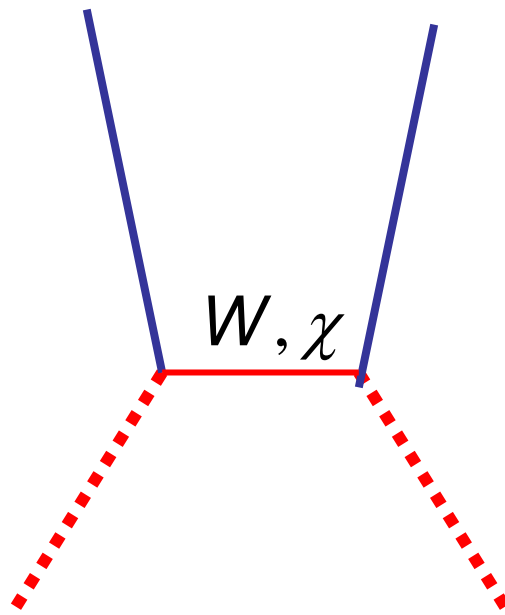


# $W^+W^- \rightarrow HH$

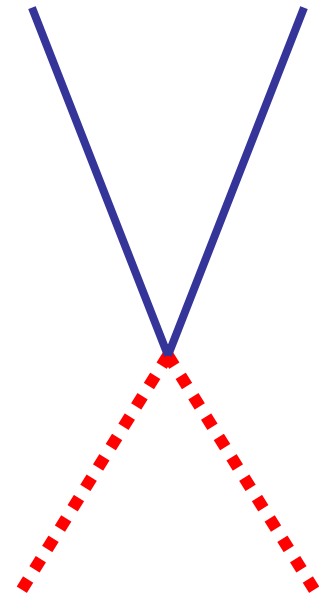
**A**nnihilation



**B**oson  
Exchange



**C**ontact  
Interaction



# $W^+W^- \rightarrow HH$

- tree ... 6 diagrams  
Annihilation, **B**oson exch., **C**ontact int.  
**B**  $\leftrightarrow$  **A**, **C** ... opposite sign
- 1-loop ... 827 diagrams  
 $\rightarrow$  13 groups
  - A** : vertex corr .HHH, WWH, AWW, ZWW
  - B** : vertex corr. WWH,  $W \chi H$
  - C** : Box

# Check : Cuv independence

	Cuv=0	Cuv=100	Cuv=10000
$M_1^{(1)}$	-2.43803300748454E-02	-2.43803300748515E-02	-2.43803300745575E-02
$M_2^{(1)}$	6.36129553455079E-03	6.36129553455044E-03	6.36129553460606E-03
$M_3^{(1)}$	7.23087719852241E-03	7.23087719852186E-03	7.23087719846840E-03
$M_4^{(1)}$	6.36129553455080E-03	6.36129553455055E-03	6.36129553462738E-03
$M_5^{(1)}$	7.23087719852242E-03	7.23087719852180E-03	7.23087719845419E-03
$M_6^{(1)}$	3.06287858179164E-02	3.06287858179168E-02	3.06287858180306E-02
$M_7^{(1)}$	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00
$M_8^{(1)}$	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00
$M_9^{(1)}$	3.27505572723379E-02	3.27505572723362E-02	3.27505572717802E-02
$M_{10}^{(1)}$	3.00769736052286E-02	3.00769736052207E-02	3.00769736047325E-02
$M_{11}^{(1)}$	3.27505572723379E-02	3.27505572723362E-02	3.27505572717802E-02
$M_{12}^{(1)}$	3.00769736052286E-02	3.00769736052209E-02	3.00769736047325E-02
$M_{13}^{(1)}$	-1.87904957771869E-01	-1.87904957771862E-01	-1.87904957771593E-01
$M_A^{(1)}$	6.24845574307105E-03	6.24845574306529E-03	6.24845574347307E-03
$M_B^{(1)}$	1.52839407221279E-01	1.52839407221259E-01	1.52839407219181E-01
$M_C^{(1)}$	-1.87904957771869E-01	-1.87904957771862E-01	-1.87904957771593E-01
Total	-2.88170948075182E-02	-2.88170948075379E-02	-2.88170948089388E-02

# Linear Gauge vs Non-Linear Gauge

NLG	Cuv=0	Cuv=100	LG	Cuv=0
$M_1^{(1)}$	-5.88235946700174E-02	-5.88235946700178E-02	$M_1^{(1)}$	-2.43803300748454E-02
$M_2^{(1)}$	-3.07618745586793E-02	3.45210584160196E-01	$M_2^{(1)}$	6.36129553455079E-03
$M_3^{(1)}$	-3.28642239568896E-02	3.47696096879355E-01	$M_3^{(1)}$	7.23087719852241E-03
$M_4^{(1)}$	-3.07618745586793E-02	3.45210584160196E-01	$M_4^{(1)}$	6.36129553455080E-03
$M_5^{(1)}$	-3.28642239568896E-02	3.47696096879354E-01	$M_5^{(1)}$	7.23087719852242E-03
$M_6^{(1)}$	2.36103152312794E-02	2.36103152312744E-02	$M_6^{(1)}$	3.06287858179164E-02
$M_7^{(1)}$	0.00000000000000E+00	-6.93889390390723E-18	$M_7^{(1)}$	0.00000000000000E+00
$M_8^{(1)}$	0.00000000000000E+00	0.00000000000000E+00	$M_8^{(1)}$	0.00000000000000E+00
$M_9^{(1)}$	1.84494128486571E-02	-3.57523045870251E-01	$M_9^{(1)}$	3.27505572723379E-02
$M_{10}^{(1)}$	2.10500413941455E-02	-3.59510279442103E-01	$M_{10}^{(1)}$	3.00769736052286E-02
$M_{11}^{(1)}$	1.84494128486572E-02	-3.57523045870251E-01	$M_{11}^{(1)}$	3.27505572723379E-02
$M_{12}^{(1)}$	2.10500413941454E-02	-3.59510279442102E-01	$M_{12}^{(1)}$	3.00769736052286E-02
$M_{13}^{(1)}$	5.46494731767579E-02	5.46494731769278E-02	$M_{13}^{(1)}$	-1.87904957771869E-01
$M_A^{(1)}$	-3.52132794387380E-02	-3.52132794387434E-02	$M_A^{(1)}$	6.24845574307105E-03
$M_B^{(1)}$	-4.82532885455325E-02	-4.82532885456059E-02	$M_B^{(1)}$	1.52839407221279E-01
$M_C^{(1)}$	5.46494731767579E-02	5.46494731769278E-02	$M_C^{(1)}$	-1.87904957771869E-01
Total	-2.88170948075125E-02	-2.88170948074217E-02	Total	-2.88170948075182E-02

# leading $m_t$ formulas

**A:**  $C_H + C_W$

$$C_H = -\frac{\alpha}{3\pi s_w^2} N_C \frac{m_t^4}{M_W^2 M_H^2}$$

**-0.11779616**

**B:**  $2C_W$

$$C_W = -\frac{\alpha}{8\pi s_w^2} N_C \frac{m_t^2}{M_W^2} \frac{2s_w^2 + 3}{12s_w^2}$$

**-0.02535196**

**C:**  $C_4$

$$C_4 = -\frac{\alpha}{8\pi s_w^2} N_C \frac{m_t^2}{M_W^2} \frac{8s_w^2 + 3}{6s_w^2}$$

**-0.07033663**

$M_W = 80.4163$

$M_Z = 91.1876$

$M_H = 120$

$m_t = 180$

$\alpha = 1/137.0359895$



$$m_t = 180 \times 10^{(n-1)} \text{ GeV } (n = 1, 2, 3, 4, 5)$$

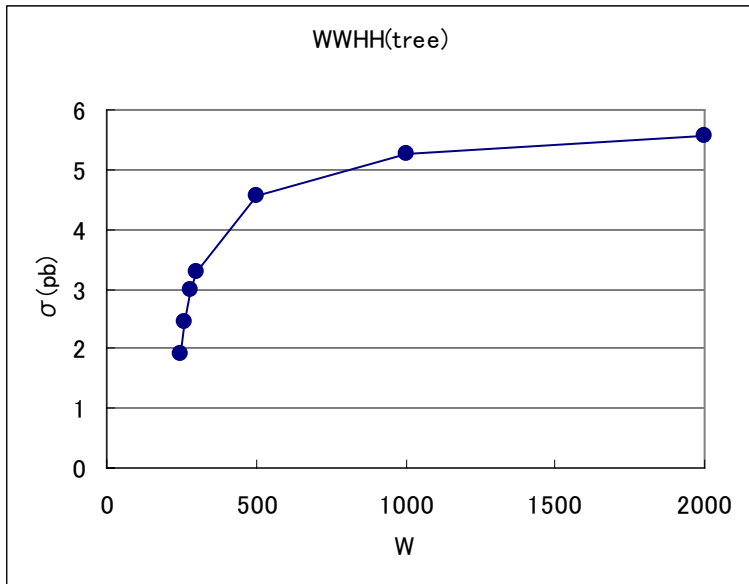
real world : NOT  $m_t \gg W, M_H, M_Z, \dots$

	$m_t = 180$	$m_t = 180 \times 10^1$	$m_t = 180 \times 10^2$	$m_t = 180 \times 10^3$	$m_t = 180 \times 10^4$
$R_1$	-3.95661868E-02	-1.16590883E+03	-1.17784063E+07	-1.17796040E+11	-1.17796160E+15
$R_2$	1.08836690E-01	-2.41343274E+00	-2.53407370E+02	-2.53518663E+04	-2.53519683E+06
$R_3$	1.09491467E-01	-2.41224707E+00	-2.53406171E+02	-2.53518651E+04	-2.53519683E+06
$R_4$	1.08836690E-01	-2.41343274E+00	-2.53407370E+02	-2.53518663E+04	-2.53519683E+06
$R_5$	1.09491467E-01	-2.41224707E+00	-2.53406171E+02	-2.53518651E+04	-2.53519683E+06
$R_6$	1.39129674E-01	-2.38545988E+00	-2.53379444E+02	-2.53518384E+04	-2.53519680E+06
$R_9$	4.49015626E-02	-2.47168199E+00	-2.53465487E+02	-2.53519244E+04	-2.53519689E+06
$R_{10}$	4.49636773E-02	-2.47197693E+00	-2.53465796E+02	-2.53519247E+04	-2.53519689E+06
$R_{11}$	4.49015626E-02	-2.47168199E+00	-2.53465487E+02	-2.53519244E+04	-2.53519689E+06
$R_{12}$	4.49636773E-02	-2.47197693E+00	-2.53465796E+02	-2.53519247E+04	-2.53519689E+06
$R_{13}$	1.03054258E-01	-6.88133668E+00	-7.03232676E+02	-7.03365171E+04	-7.03359795E+06



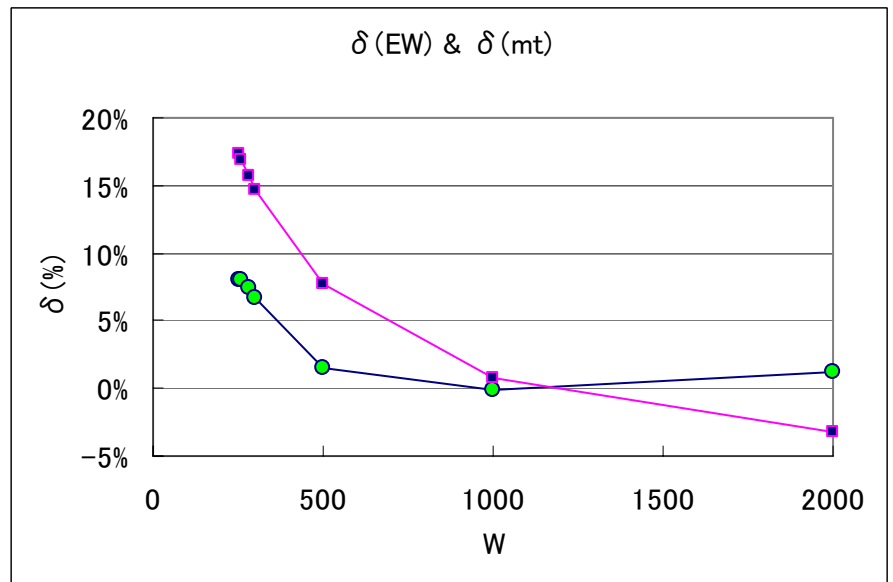
OK

*proof of the formulas and  
the performance of GRACE system*

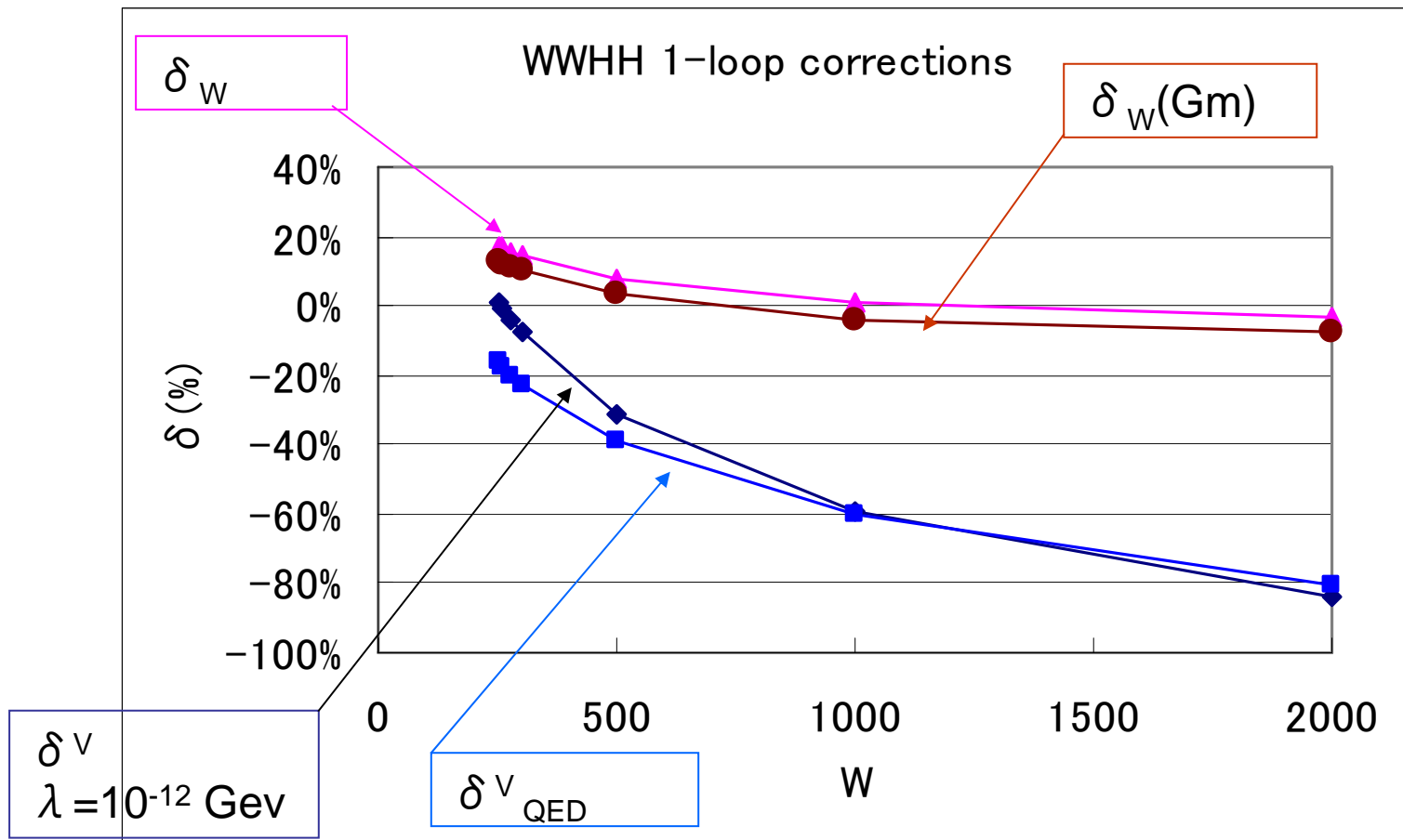


tree+leading mt correction

full EW correction (w/o QED)



# $W^+W^- \rightarrow HH$



$$e^+e^- \rightarrow \nu \nu H H$$

- number of diagrams

$\nu_e$  ... tree= 81 loop=19638

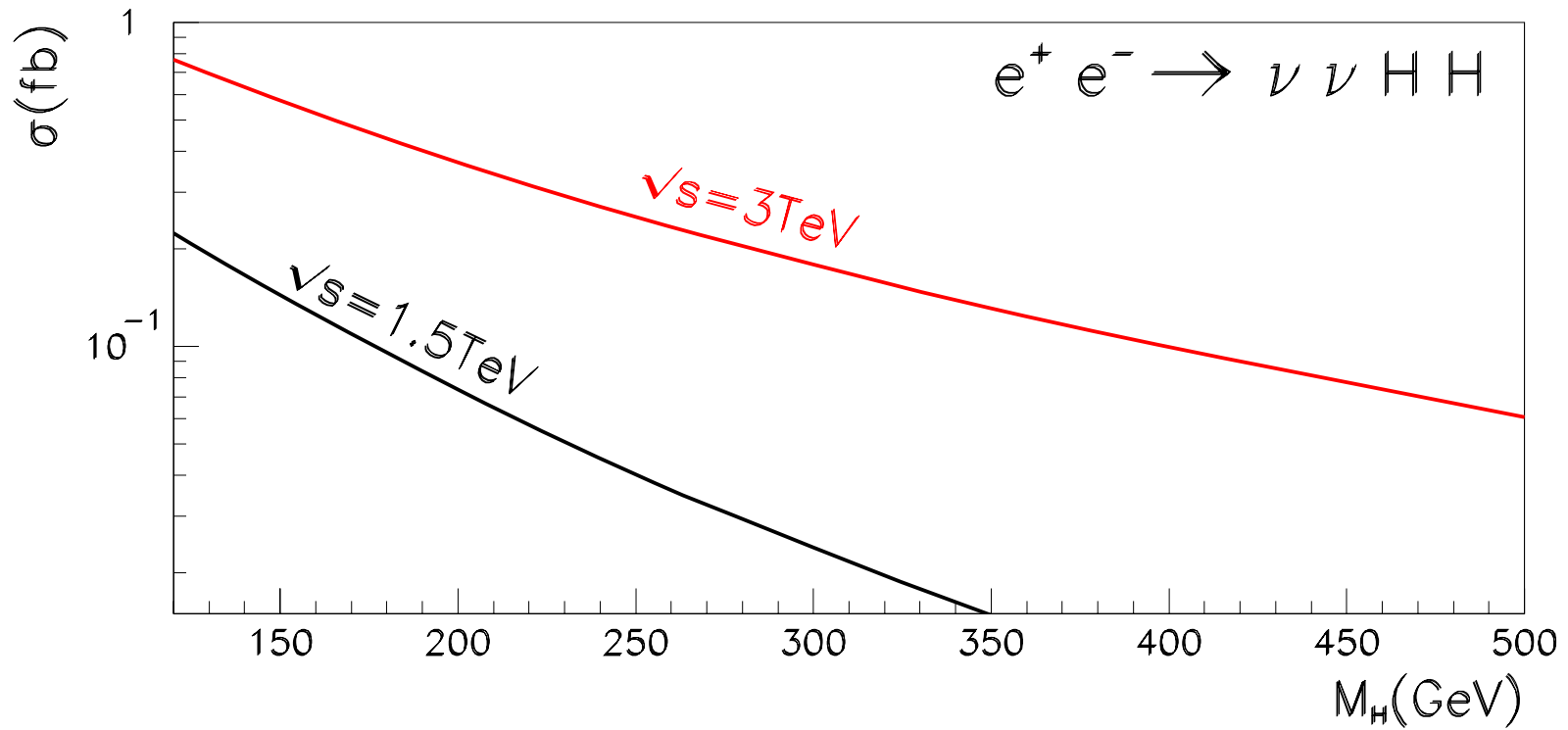
(prod. set = 12 x 3416)

50M lines of Fortran code

$\nu_{\mu}$  ... tree= 27 loop=8292

(prod. set = 6 x 1754)

$$e^+e^- \rightarrow \nu \nu HH$$

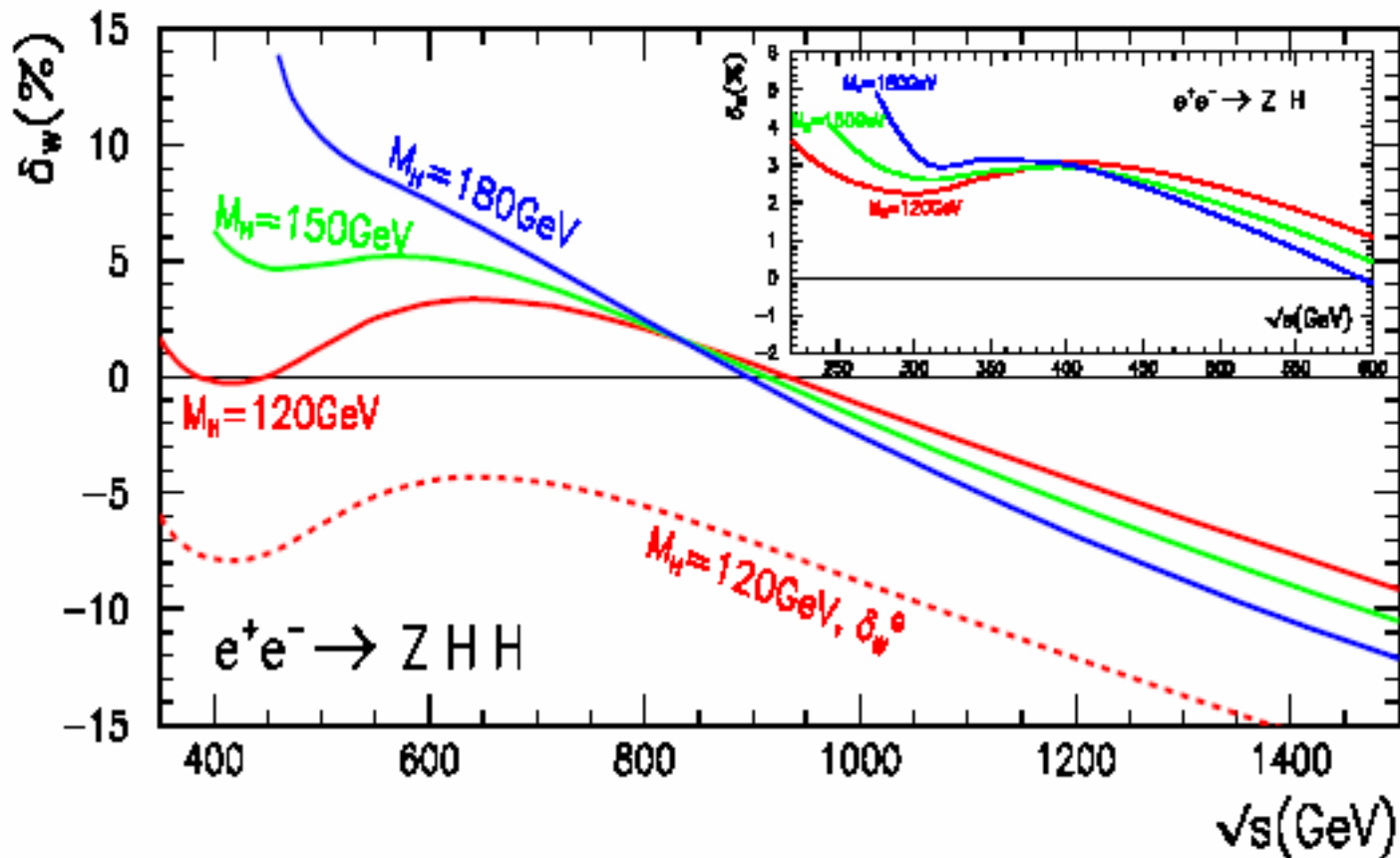
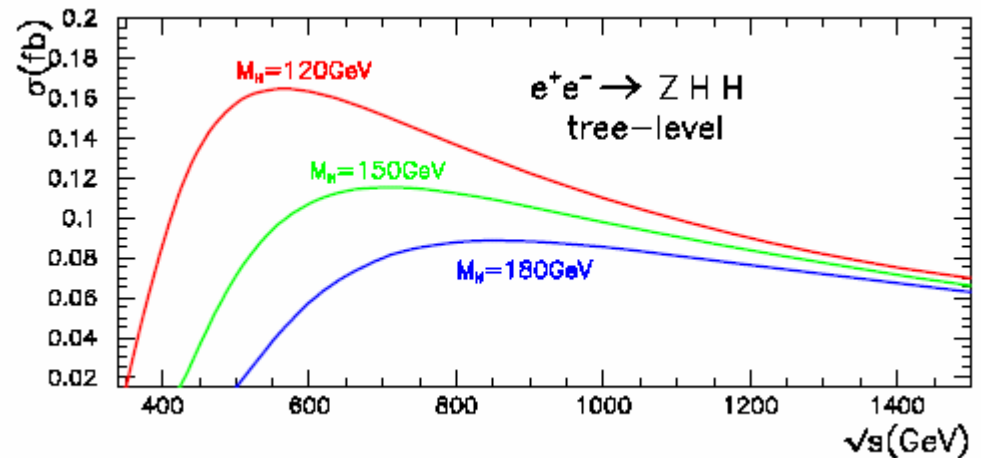


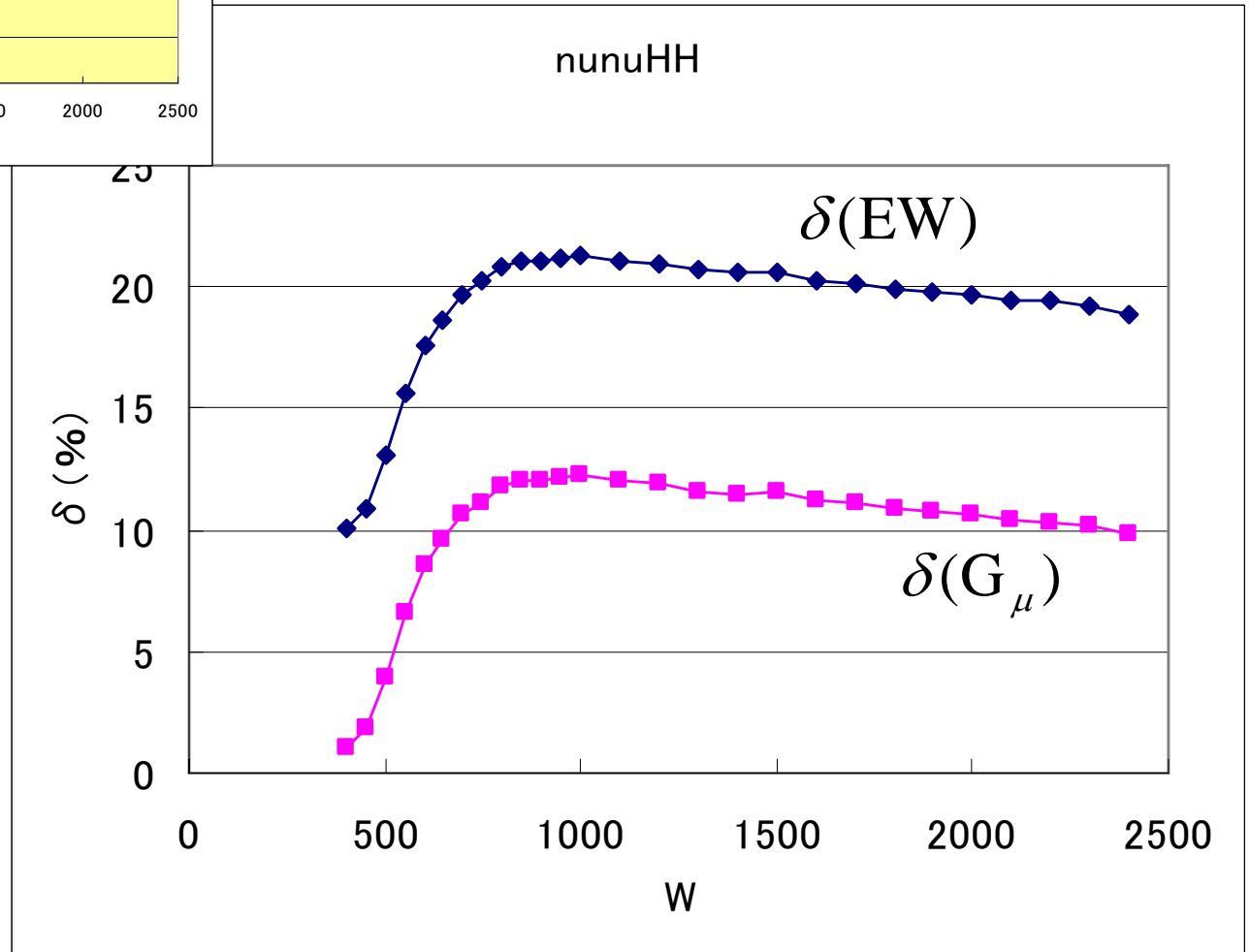
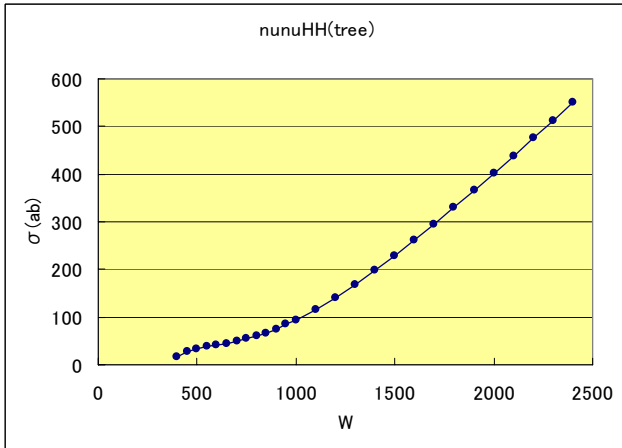
“s-channel”

$$e^+ e^- \rightarrow Z H H$$

hep-ph/030910

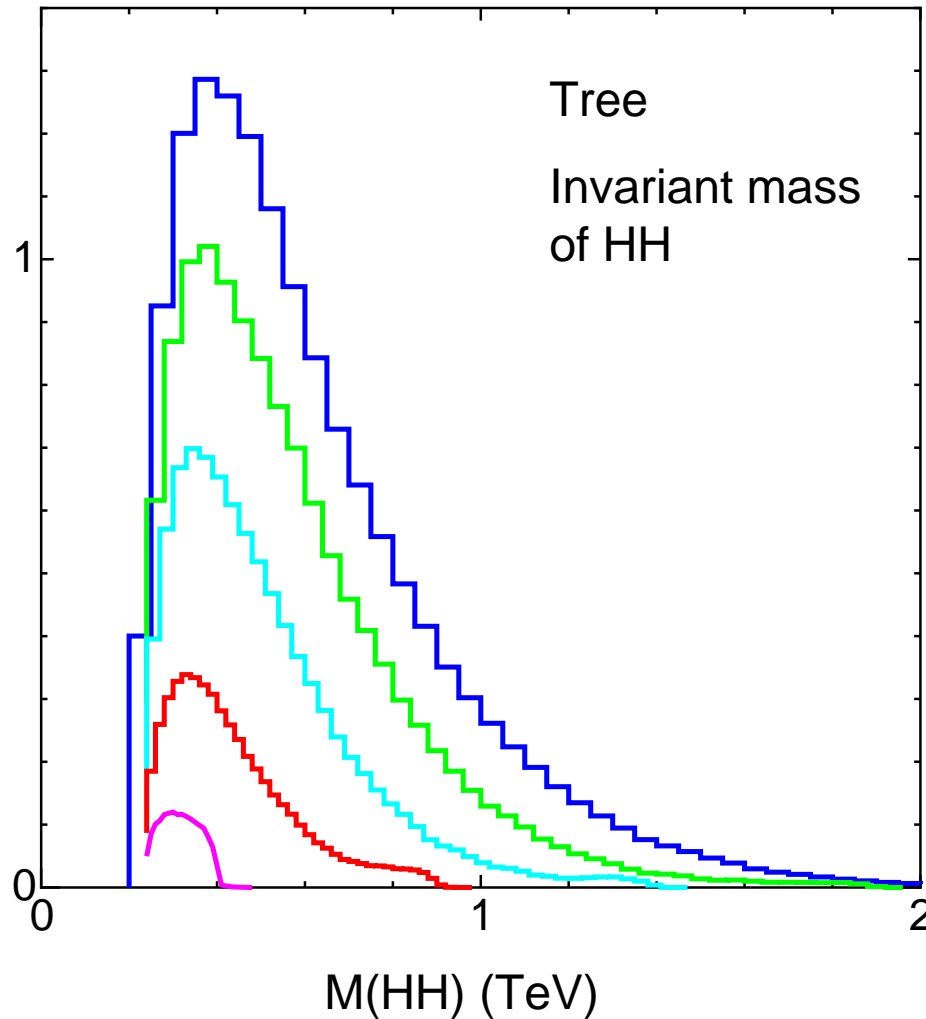
Phys.Lett.B 576(2003)152.





Full  
calculation  
 $M_H=120\text{GeV}$

$$e^+e^- \rightarrow \nu \nu HH$$



$$M_H = 120 \text{ GeV}$$

W=2.5TeV

W=2.0TeV

W=1.5TeV

W=1.0TeV

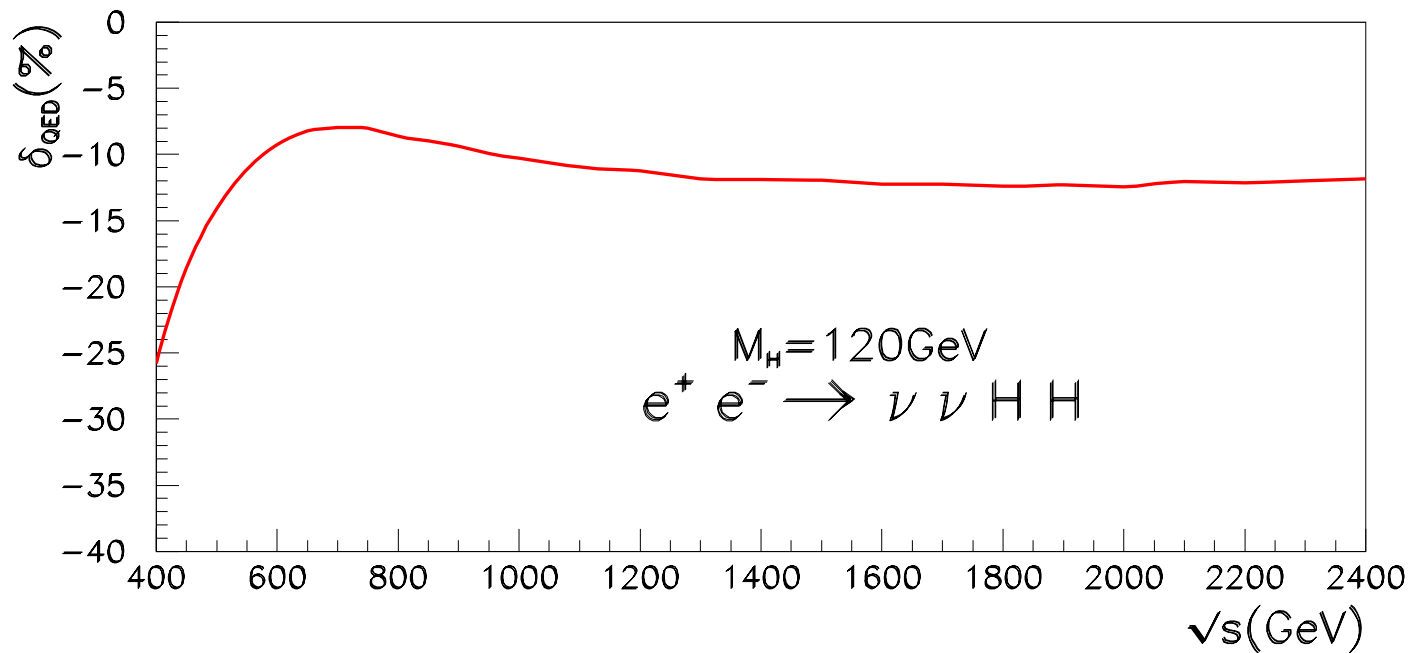
W=0.5TeV



# Summary

- Higgs Potential channel  $ee \rightarrow \nu \nu HH$  cross section and its EW correction is calculated.  $\sigma = \mathcal{O}(100 \text{ ab})$  and greater than  $ee \rightarrow ZHH$  in high-E LC region ( $\sqrt{s} \geq 1 \text{ TeV}$ ).  $\delta$  (Gmu) is  $\sim 10\%$  for  $\sqrt{s} > 700 \text{ GeV}$ . Large deviation from  $ee \rightarrow ZHH$  dominated by s-channel.
- $WW \rightarrow HH$  is studied to check the t-channel structure of  $ee \rightarrow \nu \nu HH$  and has shown validity and limitation of the leading  $m_t$  formulas.

$$e^+e^- \rightarrow \nu \nu H H$$

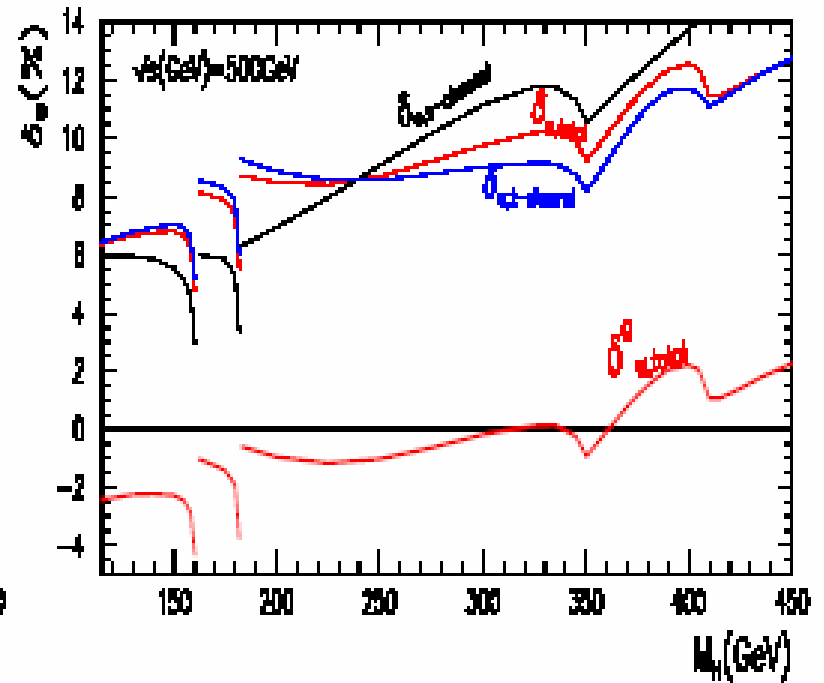
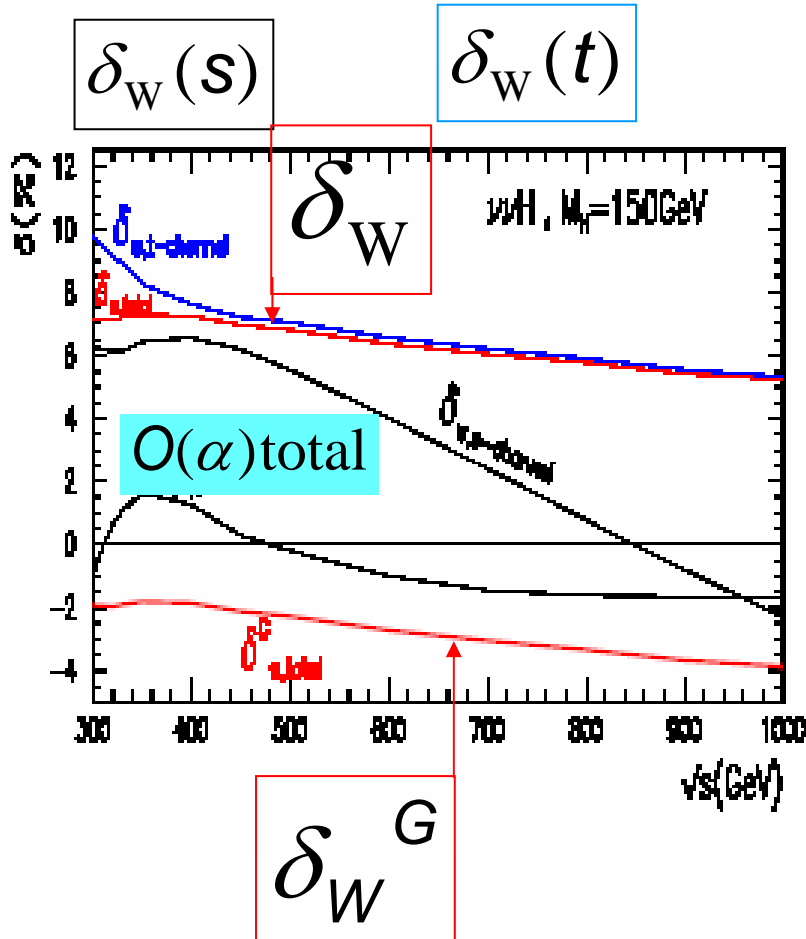


$$e^+ e^- \rightarrow \nu \bar{\nu} H$$

hep-ph/0212261  
 Phys.Lett.B 559(2003) 252.

s,t channels show different behavior  
 genuine weak correction in G-scheme is -2~ -4%

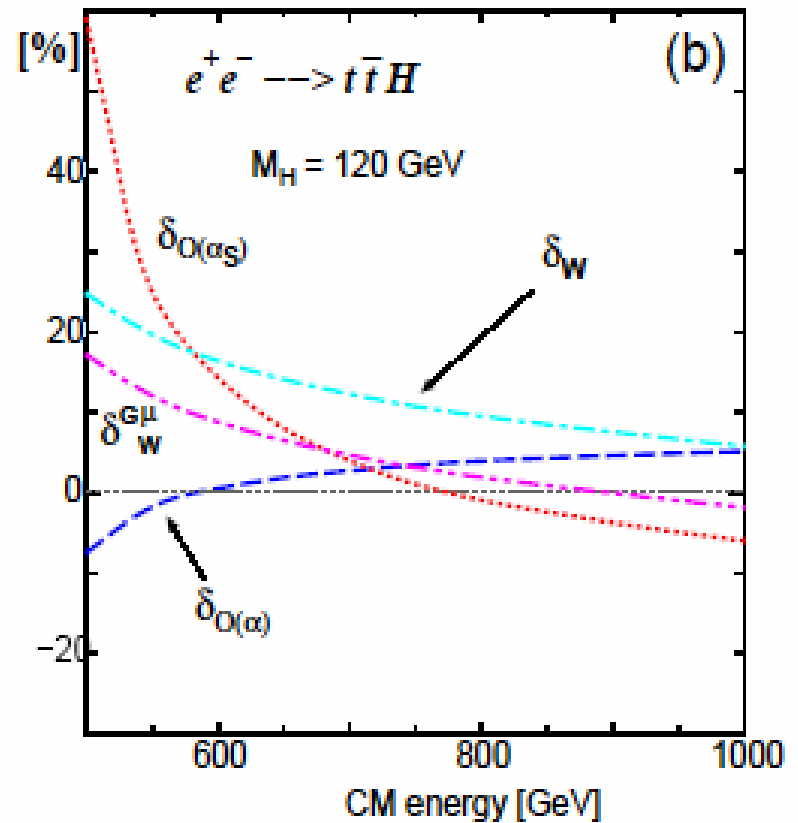
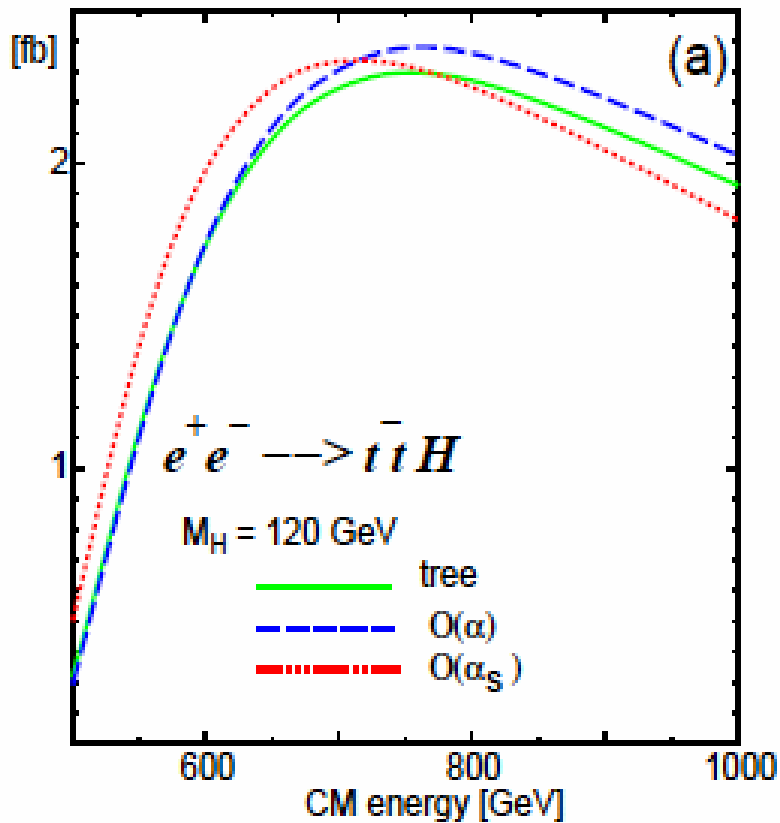
$$\delta_{\text{HWW}} = -\frac{5\alpha M_t^2}{16\pi s_W^2 M_W^2} = -1.5\%$$



$$e^+ e^- \rightarrow t\bar{t}H$$

hep-ph/0307029

Phys.Lett.B 571(2003) 163.



# system components

- Diagram generation for input process
- Amplitude/Matrix element generation
- Kinematics and Integration (efficiency)
- Event generation (efficiency & weight)
- Peripheral tools: rule generator, diagram selection, QED radiation, PDF, loop integral library, multi-process, color flow and interface for hadronization, etc.

# 5-point functions

$N$  rank M

$$I_5 = \sum G_{\mu\nu\dots\sigma} \int d\ell \frac{\ell^\mu \ell^\nu \dots \ell^\sigma}{D_0 D_1 D_2 D_3 D_4} \quad \begin{aligned} D_0 &= \ell^2 + X_0 \\ D_j &= \ell^2 + 2\ell \cdot r_j + X_j \end{aligned}$$

$$A_{ij} = r_i \cdot r_j \quad g^{\mu\nu} = r_i^\mu A_{ij}^{-1} r_j^\nu \quad \ell^2 = D_0 - X_0$$

$$\ell^\mu = r_i^\mu A_{ij}^{-1} (r_j^\nu \ell) = \frac{1}{2} r_i^\mu A_{ij}^{-1} [D_j - D_0 + X_0 - X_j]$$

$$\rightarrow N = \sum_{\alpha=0}^4 E_\alpha(\ell) D_\alpha + F \quad 1 = \sum_{\alpha=0}^4 [a_\alpha + b_{\alpha j}(\ell r_j)] D_\alpha$$

scalar 5-pt

BOX

BOX rank M-1

# $\delta$ (QED), $\delta$ (EW)

$$\sigma = \sigma_0 (1 + \delta_{QED} + \delta_W)$$

$\delta_W$  non-QED virtual corrections

$$\delta_{QED} = \delta_{QED}^V + \delta_{QED}^{soft} + \delta_{QED}^{hard}$$

phase space subtraction  $f_{LL}$  = radiator

$$\delta_{QED} = \int (d\sigma_0 \delta_{QED}^V + d\tilde{\sigma}_0 \otimes f_{LL}) + \int_{hard} (d\sigma_{1\gamma} - d\tilde{\sigma}_0 \otimes f_{LL})$$

# Non-linear gauge fixing terms

$$L_{GF} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z} (F^Z)^2 - \frac{1}{2\xi} (F^A)^2$$

$$F^\pm = \left( \partial^\mu \mp i e \tilde{\alpha} A^\mu \mp i \frac{e c_W}{s_W} \tilde{\beta} Z^\mu \right) W_\mu^\pm \quad F^A = \partial^\mu A_\mu$$

$$+ \xi_W \left( M_W \chi^\pm + \frac{e}{2s_W} \tilde{\delta} H \chi^\pm \pm i \frac{e}{2s_W} \tilde{\kappa} \chi_3 \chi^\pm \right)$$

$$F^Z = \partial^\mu Z_\mu + \xi_Z \left( M_W \chi_3 + \frac{e}{2s_W c_W} \tilde{\varepsilon} H \chi_3 \right)$$



# Samples of NLG Feynman rules

$W - W - A$

$$e[g^{\mu\nu}(p_1 - p_2)^\rho$$

$$+ (1 + \tilde{\alpha} / \xi_W)(p_3^\nu g^{\mu\rho} - p_3^\mu g^{\nu\rho})$$

$$+ (1 + \tilde{\alpha} / \xi_W)(p_2^\mu g^{\nu\rho} - p_1^\nu g^{\mu\rho})]$$

$W - \chi - A$

$$\mp ieM_W(1 - \tilde{\alpha})g^{\mu\nu}$$

modified

$\bar{c}^\mp - c^A - A - W^\pm$

$$- e^2 \tilde{\alpha} g^{\mu\nu}$$

$\bar{c}^\mp - c^A - \chi^\pm - H$

$$\mp ie^2 \frac{1}{2s_W} \tilde{\delta} \xi_W$$

ghost-ghost- vector-vector / ghost-ghost-scalar-scalar

# Non-linear gauge

- Numerator structure is the same as Feynman gauge  
→ Loop integral library
- Vertices modified
- general values → #diagrams

$$g^{\mu\nu} \quad (\text{for } \xi = 1)$$

“old” usage

→ reduce #diagram

$$\tilde{\alpha} = 1 \Rightarrow \text{no } AW\chi$$

$$\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \tilde{\kappa}$$

Check gauge invariance

→ Independence on gauge parameters