Three Gluon Decay Function by Space-like Jet Calculus[1] Beyond the Leading Order

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ABSTRACT

Three gluon decay functions in space-like gluon branching are calculated in the next-to-leading order of QCD. The calculated results satisfy crossing symmetry between space-like branching and time-like one. Some properties of the decay functions are also examined in the case of the soft gluon radiations and in that of the branching to the space-like gluon with small momentum fraction. Furthermore kinematical constraints for leading logarithmic branching due to interference effects are studied. It is pointed out that the next-to-leading order contributions are not negligible even the angular-ordering conditions are imposed into two-body branching processes.

§1. Introduction

So far, many works of experimental and theoretical studies have been devoted to jets. One of remarkable results of the studies is establishment of the parton shower[1)]. In the high energy reactions, many soft gluons and collinear partons are produced. Especially in e^+e^- annihilation, detailed studies have been made from experimental and theoretical points of views[2)]. In these processes, contributions of the next-to-leading logarithmic(NLL) order of Quantum Chromodynamics(QCD) have been included[3)].

On the contrary, the most of the works for the parton shower models in the deep inelastic scattering and those for the hadron-hadron scattering are limited to the leading logarithmic(LL) order of QCD. The higher order contributions such as the interference effects are only taken into account by the approximated forms for the soft gluon radiations in the final state and for the branching to the space-like gluon with small momentum fraction in the initial state[4)]. These works suggest that the interference contributions can be absorbed by imposing angular ordering restrictions in the LL order branching vertices. It has been also argued that these restrictions are different from those obtained in the case of the e^+e^- annihilation[5)].

In the case of e^+e^- annihilation, the properties of the angular ordering for the soft gluon radiations have also been studied by using the explicit calculations of the three-body decay functions[6)]. However the three-body decay functions of the space-like parton branching in NLL order of QCD have been only calculated for the flavor non-singlet quarks in the deep inelastic scattering[7)]. Therefore, the explicit calculations of the full NLL order of QCD for the flavor singlet sectors are desired in order to check above mentioned arguments for the interference effects as well as to evaluate actual magnitudes of the NLL order terms.

Another motivation is concerned with construction of the NLL order parton shower model, where the three-body decay functions are the most important parts of the NLL order parton shower. As discussed in Refs.3) and 7), the kinematical constraint for parton branching process is a part of the NLL contribution so that theoretically needed constraint can be derived from study of the three-body decay function.

In this paper we focus on the pure gluon decays, which dominate over other contributions in the initial state partons for the region of the small momentum fraction. We shall present the calculated results of the three gluon decay function for the space-like gluon branching in the NLL order of QCD by using space-like jet calculus proposed in Ref.7). We also examine the behaviors of the three gluon decays in some limited kinematical regions.

Contents of our paper are as following: In next section, outline of the techniques for calculation of the three-body decay will be presented. Calculated results are found in section 3. Properties of the decay functions are also discussed in this section. In final section we will make summary and will give some comments on implementation of calculated results to Monte-Carlo simulations. Practical calculations and the explicit expressions of three gluon decay function are presented in Appendices.

§2. Outline of Calculation

First we shall briefly explain about the methods of our calculations. Following the the space-like jet calculus presented in Ref.7), we calculate a process of a gluon decaying to two on mass-shell gluons and a gluon with space-like virtuality

$$g(p) \to g(k_1) + g(k_2) + g(k_3), \tag{2·1}$$

where the momenta of the mother gluon, two on mass-shell daughter gluons and one with the space-like virtuality are denoted by p, k_1, k_2 and k_3, respectively. The space-like momentum k_3 is sometimes denoted by r. We take $p^2 = k_1^2 = k_2^2 = 0$ and $r^2 = s < 0$.

The diagrams which contribute the three body decay are classified into following types according to its structure of the denominators of the squared matrix elements.
Type [A]: Two same time-like propagators($M_A \propto 1/s_{12}^2$).

Type [B]: Two same space-like propagators($M_{B1} \propto 1/s_{23}^2$ or $M_{B2} \propto 1/s_{13}^2$).

Type [C]: One time-like propagator and a space-like one ($M_{C1} \propto 1/(s_{12}s_{23})$ or $M_{C2} \propto 1/(s_{12}s_{13})$).

Type [D]: Two different space-like propagators($M_D \propto 1/(s_{13}s_{23})$).
Here we define invariants as

$$s_{ij} = (k_i + k_j)^2 \qquad (i \neq j). \tag{2·2}$$

where the relation

$$s_{12} + s_{23} + s_{13} = s \tag{2·3}$$

holds. It should be noted that $s_{12} > 0$ and $s_{13}, s_{23} < 0$.

An amplitude T_{4g} for the four gluon interaction is written by $T_{4g}s_{12}/s_{12}$, thus this contribution can be included in above four types. Corresponding diagrams are presented in Fig.1, where the four gluon interaction is not explicitly presented.

Although one loop diagrams are present in $O(\alpha_s^2)$ corrections, they only contribute to the two-body decay functions and to regularize infrared divergence of three body decay function in such a way that

$$[f(x)]_+ = f(x) - \delta(x) \int_0^1 dy f(y), \tag{2·4}$$

where the function $f(x)$ is singular at $x = 0$.

Additional contribution to the three-body decay function may come from $O(\epsilon)$ terms which annihilate the mass singular pole $1/\epsilon$ in $4+\epsilon$ dimensional integrals. Since the mass-singularity is regularized by the virtual mass M_0 in jet calculus framework,[6)] the mass singular poles do not appear in the three-body decay functions. Thus the $O(\epsilon)$ terms do not contribute to the three-body decay function. Therefore we proceed the calculation of the three-body decay functions in 4-dimensional space-time. We obtain the infrared regularized form by replacing infrared singular term $f(x)$ by $[f(x)]_+$.

In following calculation, the momentum fraction is defined as

$$x_i = \frac{k_i n}{pn} \quad , \tag{2·5}$$

where n is the light-like vector which specifies the light-cone gauge. In order to extract the collinear contributions, we introduce a projection operator **P** that acts on uncut propagator(virtual line) and extracts mass singularity from it, where the Lorentz indices are factorized by $-g_{\mu\nu}$.[6),7)]

We define the collinear contributions of the branching vertex extracted by the projection operator **P** as

$$V = g^4 \int d\Gamma \, \frac{rn}{pn} \mathbf{P} M \frac{1}{(-s)^2} \quad , \tag{2·6}$$

where g is the QCD coupling constant, M stands for the squared matrix elements summed over the polarization states for final gluons and averaged over them for initial gluon, and $d\Gamma$ is the phase space which is given by

$$d\Gamma = (2\pi)^4 \delta^{(4)}(p - k_1 - k_2 - k_3) \frac{1}{(2\pi)^3} \frac{d^3 k_1}{2k_{10}} \frac{1}{(2\pi)^3} \frac{d^3 k_2}{2k_{20}} \frac{1}{(2\pi)^4} d^4 k_3. \tag{2·7}$$

Here momenta are represented by the Sudakov variables:

$$k_i = \alpha_i n + x_i p + k_{iT} \qquad (n^2 = p^2 = nk_{iT} = pk_{iT} = 0) \tag{2·8}$$

where

$$\alpha_i = \frac{\vec{k}_{iT}^2 + k_i^2}{2x_i(pn)} \qquad (k_1^2 = k_2^2 = 0, \; k_3^2 = s) \; . \tag{2·9}$$

Using these variables, the phase space is written as

$$d\Gamma = \frac{1}{(2\pi)^6} \frac{dx_1}{2x_1} \frac{dx_2}{2x_2} d^2\vec{k}_{1T} d^2\vec{k}_{2T} ds \delta(s - r^2) \tag{2·10}$$

$$= \frac{1}{(2\pi)^6} \frac{dx_1}{2x_1} \frac{dx_3}{2x_3} d^2\vec{k}_{1T} d^2\vec{k}_{3T} ds \delta^{(+)}(k_2^2) \; . \tag{2·11}$$

The first form in Eq.(10) of $d\Gamma$ is called form-12 and the second one in Eq.(11) is called form-13. We use either form of the phase space according to the convenience of the calculation. We also define $d\tilde{\Gamma}$ by

$$d\Gamma = \frac{1}{(8\pi^2)^2} \frac{1}{4\pi^2 X} \delta(1 - x_1 - x_2 - x_3) dx_1 dx_2 dx_3 ds d\tilde{\Gamma} \tag{2·12}$$

where $X = x_1 x_2$ in form-12 and $X = x_1 x_3$ in form-13, respectively. The extracted vertex defined in Eq.(6) is given by

$$V = \left(\frac{\alpha_s}{2\pi} \right)^2 \delta(1 - x_1 - x_2 - x_3) dx_1 dx_2 dx_3 J \frac{d(-s)}{-s} \tag{2·13}$$

where $\alpha_s = g^2/4\pi$ and

$$J = \frac{1}{4\pi^2 X} d\tilde{\Gamma} \frac{rn}{pn} \mathbf{P} M \frac{-s}{(-s)^2}. \tag{2·14}$$

Since the products of k_i's are given by s's, i.e., $2k_1k_2 = s_{12}$, $2k_1k_3 = s_{13} - s$, and $2k_2k_3 = s_{23} - s$, the numerator of M is expressed in terms of x's and s, s_{12}, s_{13} and s_{23}. By use of Eq.(3), we can eliminate one of these invariants. The main part of the calculation is the integration over a phase space, $d\tilde{\Gamma}$, in order to obtain the distribution for $\delta(1 - x_1 - x_2 - x_3)dx_1dx_2dx_3$. In the integration we must treat the mass singularity, e.g., $1/s_{23} \propto 1/\vec{k}_{1T}^2$. Though technical details are different for the type of denominator, we define a vector $\vec{h}_T$ as a linear combination of $\vec{k}_{iT}$ to simplify the calculation. For instance, the term like $\vec{k}_{1T} \cdot \vec{k}_{2T}$ is to be removed inside of δ function. In calculation of the matrix elements, we used algebraic language REDUCE.[8)] Practical techniques of the calculation are presented in Appendix A.

§3. Properties of the Three Gluon Decay Function

3.1. Calculated Results

Here we present the calculated results and study the properties of the decay function for three gluon decay in the light-cone gauge. Although the amplitude for the four gluon interaction T_{4g} is included in calculation as mentioned in the previous section, this contributes only to the term $T_{4g}^* T_{4g}$ which gives constant. In order to study the relation between three-body decay function and the kinematical constraints of two-body branching, we separately show the calculated results for the types $[A] \sim [D]$. The calculated result for each type of the matrix element is written as follows:

$$J^{[A]} = \int_{M_0^2}^{(-s)} A_L \frac{ds_{12}}{s_{12}} + A_L \log\frac{y_3}{x_3} + A_N \quad , \tag{3·1}$$

$$J^{[B1]} = \int_{M_0^2}^{(-s)} B_L^{(1)} \frac{d(-s_{23})}{-s_{23}} + B_L^{(1)} \log\frac{y_1}{x_3} + B_N^{(1)} \quad , \tag{3·2}$$

$$J^{[B2]} = \int_{M_0^2}^{(-s)} B_L^{(2)} \frac{d(-s_{13})}{-s_{13}} + B_L^{(2)} \log\frac{y_2}{x_3} + B_N^{(2)} \quad , \tag{3·3}$$

$$J^{[C1]} = C_L^{(1)} \log\frac{y_1 y_3}{x_1 x_3} + C_N^{(1)} \quad , \tag{3·4}$$

$$J^{[C2]} = C_L^{(2)} \log\frac{y_2 y_3}{x_2 x_3} + C_N^{(2)} \quad , \tag{3·5}$$

$$J^{[D]} = D_L \log\frac{y_1 y_2}{x_3} + D_N \tag{3·6}$$

with $y_i = 1 - x_i (i = 1, 2, 3)$, where $O(M_0^2/(-s))$ terms are neglected. Here $J^{[j]}(j = A \sim D)$ denotes J in Eq.(14) for each type of squared matrix element M_i. M_0 is a minimum mass scale of the phase space integrations. The explicit expressions of A_L etc. are presented in Appendix B. As shown in Appendix B, the functions A_L and

B_L are the convolutions of the LL order split functions. The interference terms(types [C] and [D]) are free from mass singularity for fixed s.

Integrating over s_{ij} for the types[A] and [B] and summing over all contributions from $J^{[A]}$ to $J^{[D]}$, we obtain

$$\sum_{i=A} J^{[i]} = V_{LL}\log\frac{(-s)}{M_0^2} + V_{ggg}\ , \tag{3·7}$$

where

$$V_{LL} = A_L + B_L^{(1)} + B_L^{(2)} \tag{3·8}$$

and

$$V_{ggg} = V_L + V_N \tag{3·9}$$

with

$$\begin{aligned} V_L = {}& A_L\log\frac{y_3}{x_3} + B_L^{(1)}\log\frac{y_1}{x_3} + B_L^{(2)}\log\frac{y_2}{x_3} \\ & + C_L^{(1)}\log\frac{y_1y_3}{x_1x_3} + C_L^{(2)}\log\frac{y_2y_3}{x_2x_3} + D_L\log\frac{y_1y_2}{x_3}, \end{aligned} \tag{3·10}$$

$$V_N = A_N + B_N^{(1)} + B_N^{(2)} + C_N^{(1)} + C_N^{(2)} + D_N. \tag{3·11}$$

The first term of Eq.(21) is the contributions from the LL order vertices. The NLL order contributions are included in the three gluon decay function V_{ggg} which is constructed by the logarithmic term V_L and the non-logarithmic term V_N presented in Eqs.(24) and (25),respectively.

3.2. Crossing Symmetry

In order to verify our results, we examine the crossing relation between our result and three gluon decay function for time-like gluon decay calculated in Ref.6) for the process

$$g(q) \to g(l_1) + g(l_2) + g(l_3), \tag{3·12}$$

where the momenta of the mother parton with the time-like virtuality, three on mass-shell daughter partons are denoted by q, l_1, l_2 and l_3, respectively. For the time-like decay process, momentum fractions of partons are defined by

$$z_i = \frac{l_i n}{qn}. \tag{3·13}$$

with

$$z_1 + z_2 + z_3 = 1. \tag{3·14}$$

Replacement of the momenta

$$p \to -l_3, \quad k_3 \to -q \quad k_1 \to l_1, \quad k_2 \to l_2 \tag{3·15}$$

gives following relations for the momentum fractions of partons between space-like branching and time-like one:

$$x_1 \to -\frac{z_1}{z_3}, \qquad x_2 \to -\frac{z_2}{z_3}, \qquad x_3 \to \frac{1}{z_3}. \tag{3·16}$$

Inserting above relations in our result, we find that

$$V_{ggg}(-\frac{z_1}{z_3}, -\frac{z_2}{z_3}, \frac{1}{z_3}) \to V_{ggg}^{[T]}(z_1, z_2, z_3) + A_L(-\frac{z_1}{z_3}, -\frac{z_2}{z_3}, \frac{1}{z_3})\log(-1) \tag{3·17}$$

where $V_{ggg}^{[T]}$ denotes the three gluon decay function for the time-like gluon omitting the infrared regularization denoted by + in Ref. 6). The term with $\log(-1)$ is compensated by the analytic continuation of virtuality from $k_3^2 = s < 0$ to $q^2 > 0$ in phase space.

3.3. Numerical Results

In order to examine the numerical properties of the NLL order terms, we calculate the ratios

$$R_1 = \frac{V_{ggg}}{V_{LL}} \tag{3·18}$$

and

$$R_2 = \frac{V_L}{V_{LL}}. \tag{3·19}$$

In Fig.2(a), x_1 dependence of R_1 and R_2 are presented for $x_3 = 0.5, 10^{-1}, 10^{-2}, 10^{-3}$. In Fig.2(b), x_3 dependence of these ratios are also presented for $x_1 = 0.5, 10^{-1}, 10^{-2}, 10^{-3}$. Here R_1 and R_2 are denoted by the solid lines and the crossed symbols, respectively. Although in the most of the region $R_1 \simeq R_2$ holds, it does not mean that non-logarithmic term for each type of diagram is negligibly small compared with corresponding logarithmic term.

In Fig.3, we present the non-logarithmic contribution for each diagram at $x_3 = 0.1$. The non-logarithmic contributions for the branching diagrams(type[A] and type[B]) are canceled by those for the interference diagrams(type[C] and type[D]). Therefore the non-logarithmic contribution can be neglected only when all types of diagrams are added. This structure is held in the most of the region of the momentum fractions as shown in the Figs. 2(a) and 2(b).

3.4. Three-Body Decay Function for small x

As shown in Figs.2(a) and 2(b), the three gluon decay function becomes large for the small x_1, whereas it becomes small at small x_3. In order to understand these behaviors, we examine following two cases:
Case (i) $x_1 \ll x_2, x_3$.

Case (ii) $x_3 \ll x_1, x_2$.

The case (i) corresponds to the soft gluon radiations, while the case(ii) is the production of the space-like gluon with the small momentum fraction.

Case (i):Soft gluon radiation($x_1 \ll x_2, x_3$)

In this case, since $y_1 \simeq 1$,$y_2 \simeq x_3$ and $y_3 \simeq x_2$, the most singular term of V_{ggg} appear from the interference of type[C1] as

$$\sim -\frac{4C_A^2 K(x_3)}{x_1}\log\frac{x_2}{x_1}. \tag{3·20}$$

It has been suggested that the logarithmic contributions in the small x_1 can be absorbed by imposing further restrictions on the phase space in the two-body branching vertices.[4] For example, from Eqs.(16) and (18), absorption of the term in Eq.(34) into the two-body branching of $g(k_2 + k_3) \to g(k_2) + g(k_3)$ gives

$$\begin{aligned} &\int_{M_0^2}^{(-s)} \frac{4C_A^2 K(x_3)}{x_1}\frac{d(-s_{23})}{-s_{23}} - \frac{4C_A^2 K(x_3)}{x_1}\log\frac{x_2}{x_1} \\ &= \int_{M_0^2}^{(-s)x_1/x_2} \frac{4C_A^2 K(x_3)}{x_1}\frac{d(-s_{23})}{-s_{23}} \end{aligned} \tag{3·21}$$

since

$$B_L^{(1)} \simeq -C_L^{(1)} \simeq \frac{4C_A^2 K(x_3)}{x_1} \tag{3·22}$$

for small x_1. The phase space restriction of $-s_{23} < (-s)x_1/x_2$ in Eq.(34) is reduced to the angular ordering condition $\theta_{pk_1} < \theta_{pk_2}$ in the space like parton branching[4] for $\theta_{pk_1}, \theta_{pk_2} \ll 1$ and for $x_1 \ll x_2$, since $-s_{23} \sim x_1 E^2 \theta_{pk_1}^2$ and $-s \sim x_2 E^2 \theta_{pk_2}^2$ with $E = p^0$. In this case, the three gluon decay function should be modified by

$$V_{ggg}^M = V_{ggg} - \frac{4C_A^2 K(x_3)}{x_1}\log\frac{x_1}{x_2}. \tag{3·23}$$

V_{ggg}^M divided by V_{LL} for $x_3 = 0.1$ is presented in Fig.2(a) by dashed line. It suggests that the NLL order contributions are not negligible even the angular ordering conditions are imposed in two-body branching.

Case (ii):Small x_3 region($x_3 \ll x_1, x_2$)

In this case, since $y_1 \simeq x_2$,$y_2 \simeq x_1$ and $y_3 \simeq 1$, V_{ggg} is approximated by

$$\begin{aligned} V_{ggg} \simeq\ & A_L\log\frac{1}{x_3} + B_L^{(1)}\log\frac{x_2}{x_3} + B_L^{(2)}\log\frac{x_1}{x_3} \\ & + C_L^{(1)}\log\frac{x_2}{x_1 x_3} + C_L^{(2)}\log\frac{x_1}{x_2 x_3} + D_L\log\frac{x_1 x_2}{x_3}. \end{aligned} \tag{3·24}$$

From the Appendix B, the coefficients of the logarithmic terms are approximated by

$$A_L \simeq B_L^{(1)} \simeq B_L^{(2)} \simeq -C_L^{(1)} \simeq -C_L^{(2)} \simeq -D_L \simeq \frac{4C_A^2 K(x_1)}{x_3}. \qquad (3{\cdot}25)$$

Therefore $O(x_3^{-1})\log x_3$ terms are canceled in this limit. This is the reason why absolute value of V_{ggg} becomes small for the small x_3 as seen in Fig.2(b).

§4. Summary and Comments

We have calculated the three-body decay function of a gluon in the initial state which decays to one gluon with the space-like virtuality and two on-mass shell gluons. We have also presented the explicit expressions and some techniques for the phase space integrations. The calculated results satisfy crossing symmetry between the space-like branching and the time-like one.

We also studied the properties of the three gluon decays for the soft gluon radiations and those for the gluon branching with the small momentum fraction in the initial state. For the small momentum fraction x of the out-going gluon, the decay function behaves like $\sim (\log x)/x$ which can be absorbed by imposing further restrictions of the phase space in the two-body branching vertices, which leads to the angular ordering for the angle between out-going gluon and the gluon in the initial state.

Although the singular contributions are suppressed at small momentum fraction of produced gluons by imposing angular ordering, the NLL contributions still remain, which are usually neglected in Monte-Carlo simulations. They should be taken into account as NLL order corrections even the angular ordering conditions are imposed.

On the other hand, for the small momentum fraction of the space-like gluon, the $(\log x)/x$ terms are canceled each other. Thus the three gluon decay function becomes small.

In approximate approaches the logarithmic contributions due to the interference terms are separately absorbed to each of the two-body branching. However these arguments are meaningful only for the case where the non-logarithmic terms are negligibly small. We found that the non-logarithmic term for each type of diagram is sizable but they are canceled each other. The non-logarithmic contributions can be neglected only after all types of diagrams are added.

Finally we shall comment on the implementation of our results to Monte-Carlo simulations. Although the modified three-gluon decay function V_{ggg}^M has no large logarithmic correction like $\sim \log x/x$ for small x, this gives negative contribution in some region. In parton shower models, the momentum fractions of branching partons are generated according to the decay functions. Thus they must be positive. In order to obtain positive probability, further modifications are needed to the three-body decay functions, which necessarily lead to the change of the kinematical constraints for the two-body branching processes. We will discuss this point in future paper.

It may be important to proceed exact calculation for other decay functions of the parton branching in the initial state in NLL order of QCD and to check the accuracy of the approximations in the limited kinematical regions. Furthermore these knowledge may be useful to construct NLL order parton shower models. In future papers, we will present NLL terms of other processes which contribute to the singlet sector and will discuss about implementation of our calculations to Monte-Carlo simulations.

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Appendix A

We present the practical calculation for the type [A] and type[C1]. Calculation for other terms can be performed by similar manner.

A.1 Calculation of Type[A]

In type[A], we use form-13 for the phase space in which there is $\delta(k_2^2)$. This constraint is expressed as

$$k_2^2 = \frac{x_2}{x_3}(-s) - \frac{y_3}{x_1}\vec{h}_T^2 - \frac{x_2}{x_3 y_3}\vec{k}_{3T}^2 = 0 \tag{A.1}$$

where $\vec{k}_{1T}$ is replaced by a new vector $\vec{h}_T$ which is defined by

$$\vec{h}_T = \vec{k}_{1T} + \frac{x_1}{y_3}\vec{k}_{3T} \tag{A.2}$$

with $y_i = 1 - x_i (i = 1, 2, 3)$.

Then we integrate the phase space

$$d\tilde{\Gamma} = \frac{\pi}{2} d(\vec{k}_{3T}^2) d\phi d(\vec{h}_T^2)\delta(k_2^2) \ . \tag{A.3}$$

¿From Eq.(A.1) and

$$\vec{k}_{3T}^2 = y_3(-s) - x_3 s_{12} \tag{A.4}$$

$\vec{h}_T^2$ is written as

$$\vec{h}_T^2 = \frac{x_1 x_2}{y_3^2} s_{12}. \tag{A.5}$$

The integral in $\vec{h}_T^2$ is trivial and $\vec{k}_{3T}^2$ is replaced by s_{12} using Eq.(A.4). The boundary of integral is determined as

$$0 < s_{12} < \frac{y_3}{x_3}(-s) \tag{A.6}$$

since $\vec{h}_T^2 > 0$ and $\vec{k}_{3T}^2 > 0$ in Eqs.(A.4) and (A.5). Thus we have

$$\int d\tilde{\Gamma} = \frac{\pi^2 x_1 x_3}{y_3} \int_{M_0^2}^{(-s)y_3/x_3} d(s_{12}) \frac{d\phi}{2\pi} \ . \tag{A.7}$$

Next we notice that the numerator is expressed by the quadratic form of invariants. Using Eq.(3) in text we can eliminate s_{13} and the integrand of J becomes

$$\frac{rn}{pn} \mathbf{P} M = \frac{F_1 s^2 + F_2 s_{12}^2 + F_3 s_{12} s + F_4 s s_{23} + F_5 s_{12} s_{23} + F_6 s_{23}^2}{s_{12}^2} \tag{A.8}$$

where F's are functions of x's. Next ϕ integration is considered since the variable s_{23} depends on ϕ. It is

$$s_{23} = (p - k_1)^2 = -\frac{1}{x_1}\vec{h}_T^2 - \frac{x_1}{y_3^2}\vec{k}_{3T}^2 + \frac{2}{y_3} \mid \vec{h}_T \mid\mid \vec{k}_{3T} \mid \cos\phi. \tag{A.9}$$

By the integration over the azimuthal angle ϕ, the term $|\vec{h}_T||\vec{k}_{3T}|\cos\phi$ drops. Taking Eq.(A.4) into account, s_{23} and s_{23}^2 in the numerator become

$$s_{23} \to \frac{-x_2 + x_1x_3}{y_3^2}s_{12} - \frac{x_1}{y_3}(-s)\ ,$$
$$s_{23}^2 \to [\frac{-x_2 + x_1x_3}{y_3^2}s_{12} - \frac{x_1}{y_3}(-s)]^2 + \frac{2x_1x_2}{y_3^4}[y_3(-s) - x_3s_{12}]s_{12}. \quad (A.10)$$

Therefore the integration of M over ϕ gives us

$$\int \frac{d\phi}{2\pi}\frac{rn}{pn}\sum M = \frac{G_1s^2 + G_2s_{12}^2 + G_3s_{12}s}{s_{12}^2} \quad (A.11)$$

where G's are functions of x's. Since the mass singularity is of order of the logarithm, the relation

$$G_1 = 0 \quad (A.12)$$

should be held. This fact is useful for a check of the calculation. Here G_i is obtained by algebraic calculations. Substituting Eqs.(A.7) and (A.11) into Eq.(14) in text we obtain the final result

$$\begin{aligned} J &= \frac{1}{4y_3}\int_{M_0^2}^{(-s)y_3/x_3}(-G_3\frac{ds_{12}}{s_{12}} + G_2\frac{ds_{12}}{-s}) \\ &= \frac{1}{4y_3}\int_{M_0^2}^{(-s)}(-G_3)\frac{ds_{12}}{s_{12}} + \frac{1}{4y_3}(-G_3)\log\frac{y_3}{x_3} \\ &+ \frac{1}{4x_3}G_2 + O(\frac{M_0^2}{-s}). \end{aligned} \quad (A.13)$$

The first term is the LL contribution and the second and the third terms contribute to the three-body decay functions which we write as

$$\frac{1}{4y_3}(-G_3)\log\frac{y_3}{x_3} + \frac{1}{4x_3}G_2 = A_L\log\frac{y_3}{x_3} + A_N \quad (A.14)$$

with

$$A_L = \frac{-G_3}{4y_3} \quad \text{and} \quad A_N = \frac{G_2}{4x_3}. \quad (A.15)$$

The $O(M_0^2/(-s))$ term is a part of the two-body decay function, thus we neglect this term in three-body decay function. Contributions from type[B1] and [B2] are also calculated by similar method.

A.2 Calculation of Type C1

For interference terms such as types[C] and [D], the logarithmic terms $\log(-s/M_0^2)$ do not appear from integration over invariant s_{ij}, because these contributions are free from the mass singularity for fixed s. We perform the calculation of the interference diagram[C1], where both of the space-like and the time-like virtual partons appear. While in the types[A] and [B] where we can take both the denominator and the constraint(δ-function) to be ϕ-independent by the rearrangement of transverse vectors, such procedure is not possible in the interference case, so that the computation is more complicated.

After eliminating s_{13} by Eq.(3) in the text, the integrand of J becomes

$$\frac{rn}{pn}\mathbf{P}M = \frac{F_1 s_{12} s_{23} + F_2 s^2 + F_3 s s_{12} + F_4 s s_{23} + F_5 s_{12}^2 + F_6 s_{23}^2}{s_{12} s_{23}} . \qquad (A.16)$$

In order to process the last two terms, we define

$$K_1 = \int d\tilde{\Gamma}\frac{s_{12}^2}{s_{12}s_{23}} = \int d\tilde{\Gamma}\frac{s_{12}}{s_{23}} \qquad (A.17)$$

and

$$K_2 = \int d\tilde{\Gamma}\frac{s_{23}^2}{s_{12}s_{23}} = \int d\tilde{\Gamma}\frac{s_{23}}{s_{12}}. \qquad (A.18)$$

The variable s_{12} depends on ϕ which is

$$s_{12} = (p-k_3)^2 = \frac{x_1}{x_2}\vec{h}_T^2 + \frac{x_2}{x_1 y_1^2}\vec{k}_{1T}^2 + \frac{2}{y_1} \mid \vec{k}_{1T} \mid\mid \vec{h}_T \mid cos\phi \qquad (A.19)$$

with

$$\vec{h}_T = \vec{k}_{3T} + \frac{x_3}{y_1}\vec{k}_{1T}. \qquad (A.20)$$

Note that k_2^2 in delta function is independent of ϕ for $\vec{h}_T$ defined in Eq.(A.20). By the integration over the azimuthal angle ϕ, the term $\mid \vec{k}_{1T} \mid\mid \vec{h}_T \mid \cos\phi$ drops. Taking Eqs.(A.19) into account, s_{12} in the numerator of Eq.(A.17) become

$$s_{12} \to \frac{x_1}{y_1}(-s) + \frac{x_2 - x_1 x_3}{y_1^2}(-s_{23}). \qquad (A.21)$$

Using Eq.(A.21), we can modify K_1 as

$$K_1 = \int d\tilde{\Gamma}\frac{s_{12}}{s_{23}} \to \int d\tilde{\Gamma}\frac{1}{s_{23}}\left(\frac{x_1}{y_1}(-s) + \frac{x_2 - x_1 x_3}{y_1^2}(-s_{23})\right) . \qquad (A.22)$$

similarly, K_2 can be modified by using Eq.(A.10) in type[A] as

$$K_2 = \int d\tilde{\Gamma}\frac{s_{23}}{s_{12}} \to \int d\tilde{\Gamma}\frac{1}{s_{12}}\left(-\frac{x_1}{y_3}(-s) + \frac{-x_2 + x_1 x_3}{y_3^2}s_{12}\right) . \qquad (A.23)$$

By the above modification, we can write

$$\int d\tilde{\Gamma}\frac{rn}{pn}\mathbf{P}M = \int d\tilde{\Gamma}\frac{G_1 s_{12}s_{23} + G_2 s^2 + G_3 s s_{12} + G_4 s s_{23}}{s_{12}s_{23}} \qquad (A.24)$$

where G's are functions of x's.

The interference term must be free from mass-singularity which may occur at $s_{12} = 0$ or $s_{23} = 0$. The term $G_1 s_{12} s_{23}$ in the numerator is of course free from mass-singularity. Below we present that there is another non-trivial form which is mass-singularity free. By practical calculation we find that G's satisfy following relations:

$$G_3 = \frac{y_1}{x_1} G_2, \quad G_4 = -\frac{y_3}{x_1} G_2 \qquad (A.25).$$

Therefore the integration in Eq.(A.24) is written by

$$\int d\tilde{\Gamma}[G_1 + \frac{G_2 s}{s_{12}s_{23}}(\frac{y_1}{x_1}s_{12} - \frac{y_3}{x_1}s_{23} + s)]. \qquad (A.26)$$

Then we calculate the following integrals:

$$K_0 = \int d\tilde{\Gamma},$$
$$K_3 = \int d\tilde{\Gamma}\frac{s}{s_{12}s_{23}}\left(s + \frac{y_1}{x_1}s_{12} + \frac{y_3}{x_1}(-s_{23})\right). \qquad (A.27)$$

Here we use form-13 for the phase space integration. replacing $\vec{k}_{3T}$ by new vector $\vec{h}'_T$ defined by

$$\vec{h}'_T = \frac{x_1 y_1}{x_2}\left(\vec{k}_{3T} + \frac{x_3}{y_1}\vec{k}_{1T}\right) \qquad (A.28)$$

the constraint inside the δ function is expressed as

$$k_2^2 = \frac{x_2}{x_3}(-s) - \frac{x_2^2}{x_1^2 y_1 x_3}\vec{h}'^2_T - \frac{x_2}{x_1 y_1}\vec{k}^2_{1T} = 0 \qquad (A.29)$$

Then we integrate the phase space

$$d\tilde{\Gamma} = \left(\frac{\pi}{2}\right)^3 \left(\frac{x_2}{x_1 y_1}\right)^2 d(\vec{k}^2_{1T})d\phi d(\vec{h}'^2_T)\delta(k_2^2). \qquad (A.30)$$

¿From Eq.(A.29) and

$$-s_{23} = \frac{\vec{k}^2_{1T}}{x_1}, \qquad (A.31)$$

the boundary of integral is given by

$$0 < -s_{23} < \frac{y_1}{x_3}(-s) \qquad (A.32)$$

due to $\vec{h}'^{\,2}_T > 0$ and $\vec{k}^2_{1T} > 0$. Thus integrating over $\vec{h}'^{\,2}_T$ we have

$$d\tilde{\Gamma} = \frac{\pi^2 x_3 x_1}{y_1}\int_{M_0^2}^{(-s)y_1/x_3} d(-s_{23})\frac{d\phi}{2\pi}\,. \qquad (A.33)$$

The first one in Eq.(A.27) is trivial from Eq.(A.33) as

$$K_0 = \pi^2 x_1(-s). \qquad (A.34)$$

For K_3, using Eqs.(A.28),(A.29) and (A.31), integration over ϕ is written by

$$\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{1}{s_{12}}(\frac{y_1}{x_1}s_{12} - \frac{y_3}{x_1}s_{23} + s)$$

$$= \frac{2y_1}{x_1} \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{\vec{k}_{1T}^2 + \vec{k}_{1T}\vec{h}'_T}{(\vec{k}_{1T} + \vec{h}'_T)^2}. \qquad (A.35)$$

Using the formula

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \frac{1}{(\vec{k}_{1T} + \vec{h}'_T)^2} = \frac{1}{\mid \vec{k}_{1T}^2 - \vec{h}'^{\,2}_T \mid}, \qquad (A.36)$$

we have

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \frac{\vec{k}_{1T}^2 + \vec{h}'_T\vec{k}_{1T}}{(\vec{k}_{1T} + \vec{h}'_T)^2} = \theta(\vec{k}_{1T}^2 - \vec{h}'^{\,2}_T) \qquad (A.37)$$

where θ is the step function which cuts the singularity at $\vec{k}_{1T}^2 = 0$. ¿From Eqs.(A.29) and (A.31)

$$\vec{k}_{1T}^2 > \vec{h}'^{\,2}_T \quad \rightarrow \quad -s_{23} > \frac{x_1}{y_3}(-s). \qquad (A.38)$$

Therefore the integral K_3 is written by

$$K_3 = 2\pi^2 x_3 \int_{(-s)x_1/y_3}^{(-s)y_1/x_3} \frac{d(-s_{23})}{(-s_{23})}(-s) = 2\pi^2 x_3(-s) \log \frac{y_1 y_3}{x_1 x_3}. \qquad (A.39)$$

Finally we obtain the result

$$J = \frac{1}{4\pi^2 x_1 x_3(-s)}(K_0 G_1 + K_3 G_2)$$
$$= \frac{1}{4x_3} G_1 + \frac{1}{2x_1} \log \frac{y_1 y_3}{x_1 x_3} G_2 \; . \qquad (A.40)$$

In the text, J is written by using

$$C_L^{(1)} = \frac{G_2}{2x_1} \quad \text{and} \quad C_N^{(1)} = \frac{G_1}{4x_3}. \qquad (A.41)$$

Similar method can be used in calculation of type [C2] and [D].

Appendix B

In this appendix we present the calculated results of A_L etc. Here $K(x_i) = (1 - x_i y_i)^2 / x_i y_i$ with $y_i = 1 - x_i$. $C_A = 3$ is the color factor.
Type[A]:

$$
\begin{aligned}
A_L &= 4C_A^2 \frac{K(x_3)}{y_3} K(\frac{x_1}{y_3}) \\
A_N &= 4C_A^2 [-\frac{K(x_3)}{y_3} K(\frac{x_1}{y_3}) + \frac{(1+x_3)^2 (x_1 - x_2)^2}{8 y_3^4}] + \frac{9}{4} C_A^2,
\end{aligned}
$$

where the last term of A_N comes from the four gluon interaction $T_{4g}^* T_{4g}$.

Type[B1]:

$$
\begin{aligned}
B_L^{(1)} &= 4C_A^2 \frac{K(x_1)}{y_1} K(\frac{x_3}{y_1}) \\
B_N^{(1)} &= 4C_A^2 [-\frac{K(x_1)}{y_1} K(\frac{x_3}{y_1}) + \frac{(1+x_1)^2 (x_2 - x_3)^2}{8 y_1^4}]
\end{aligned}
$$

Type[B2]:

$$
\begin{aligned}
B_L^{(2)} &= (x_1 \ \leftrightarrow \ x_2 \ ) \ \text{ in } \ B_L^{(1)} \\
B_N^{(2)} &= (x_1 \ \leftrightarrow \ x_2 \ ) \ \text{ in } \ B_N^{(1)}
\end{aligned}
$$

Type[C1]:

$$
\begin{aligned}
C_L^{(1)} &= 2C_A^2 [\frac{x_3^3}{x_1} (\frac{1}{y_3} + \frac{1}{x_2 y_1}) - \frac{K(x_3)}{x_1} - \frac{y_2^2}{x_2} K(\frac{x_3}{y_2}) \\
&\quad - \frac{y_1}{x_1} K(\frac{x_3}{y_1}) + \frac{x_1(x_3 - x_1)}{y_1 y_3} K(\frac{x_3}{x_1}) - \frac{3x_1}{x_3}] \\
C_N^{(1)} &= C_A^2 [f_N(-1, x_1, x_2) + f_N(x_2, x_1, -1)]
\end{aligned}
$$

Type[C2]:

$$
\begin{aligned}
C_L^{(2)} &= (x_1 \ \leftrightarrow \ x_2 \ ) \ \text{ in } \ C_L^{(1)} \\
C_N^{(2)} &= (x_1 \ \leftrightarrow \ x_2 \ ) \ \text{ in } \ C_N^{(1)}
\end{aligned}
$$

Type[D]:

$$
\begin{aligned}
D_L &= 2C_A^2 [-\frac{x_3^3}{y_1 x_2} + y_2 K(\frac{x_3}{y_2}) - \frac{K(x_3)}{x_1} + \frac{K(-x_3)}{y_2} + \frac{3}{2x_3}] \\
&\quad + (x_1 \ \leftrightarrow \ x_2) \\
D_N &= C_A^2 [f_N(x_1, -1, x_2) + f_N(x_2, -1, x_1)]
\end{aligned}
$$

Here the function f_N is defined by

$$f_N(a_i, a_j, a_k) = \frac{5}{8} - \frac{2x_3^3}{a_i a_j a_k} - \frac{4x_3^2 + x_3 a_k + 4a_k^2}{(x_3 + a_i)^2} + \frac{7}{2}\frac{x_3 - a_k}{x_3 + a_i} + \frac{8x_3^2 + 4x_3 a_k + 4a_k^2}{a_i x_3} + \frac{4a_i}{x_3} + \frac{4x_3^2 - 2a_i a_k}{x_3 a_j}.$$

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FIGURE CAPTIONS

Fig. 1 Diagrams for the squared matrix elements which contribute to the decay function V_{ggg}. The solid lines denote the gluon lines. The crossed symbol is the projection operator **P** which extracts the collinear contributions.

Fig. 2 The behavior of the ratios R_1 and R_2 defined by Eqs.(32) and (33) in the text: (a) The x_1 dependence for $x_3 = 0.5, 10^{-1}, 10^{-2}$ and 10^{-3}. The solid lines and the crossed symbols denote R_1 and R_2, respectively. The dashed line denotes the modified three gluon decay function V_{ggg}^M obtained in Eq.(37) in the text divided by V_{LL} for $x_3 = 0.1$ (b) The x_3 dependence for $x_1 = 0.5, 10^{-1}, 10^{-2}$ and 10^{-3}. Notations are the same as those for (a).

Fig. 3 The x_1 dependence of the non-logarithmic terms at $x_3 = 0.1$. Here A_N etc. are defined by Eqs.(15) $\sim$ (20) in the text.

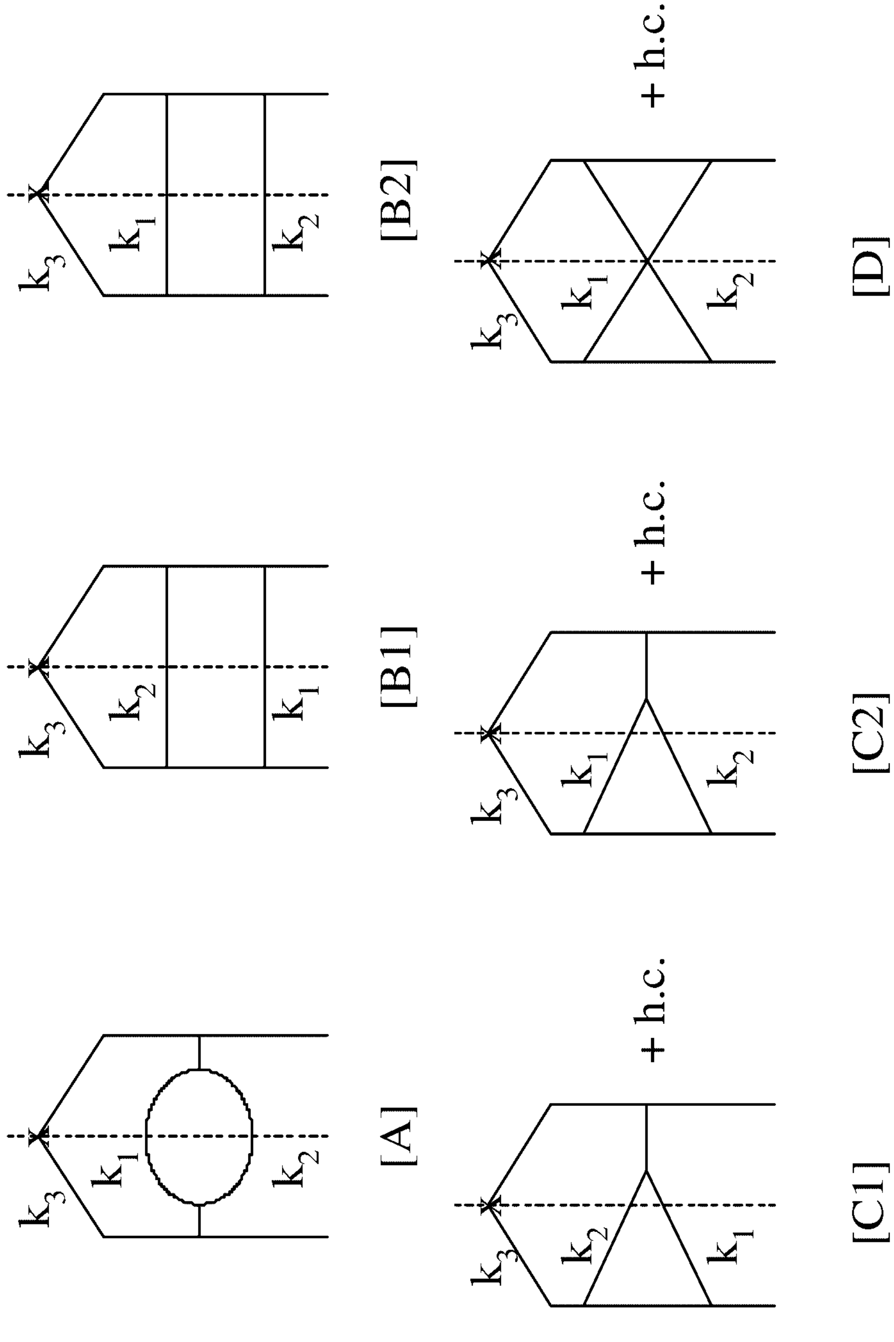

Fig. 1. Fig.1

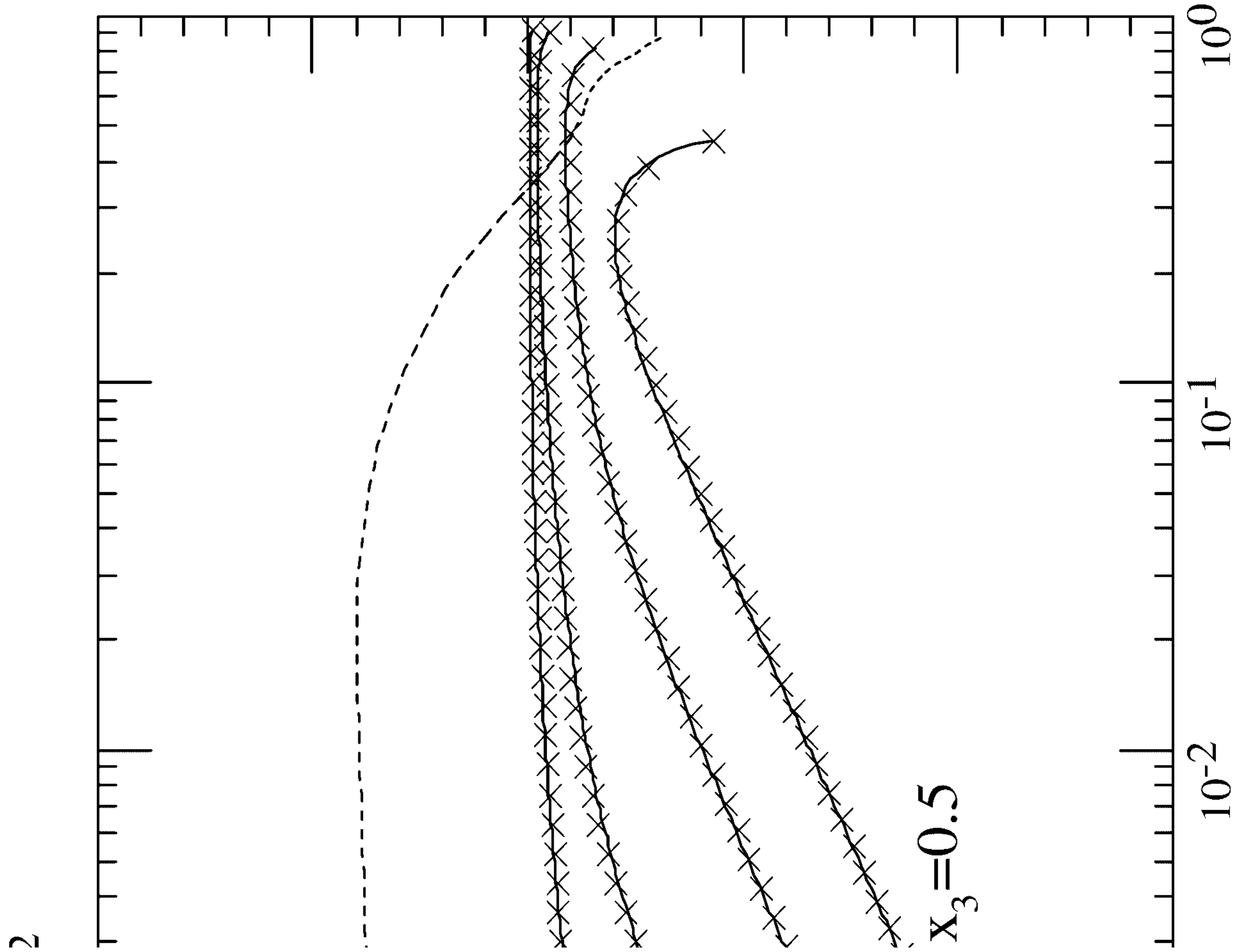

Fig. 2. Fig.2(a)

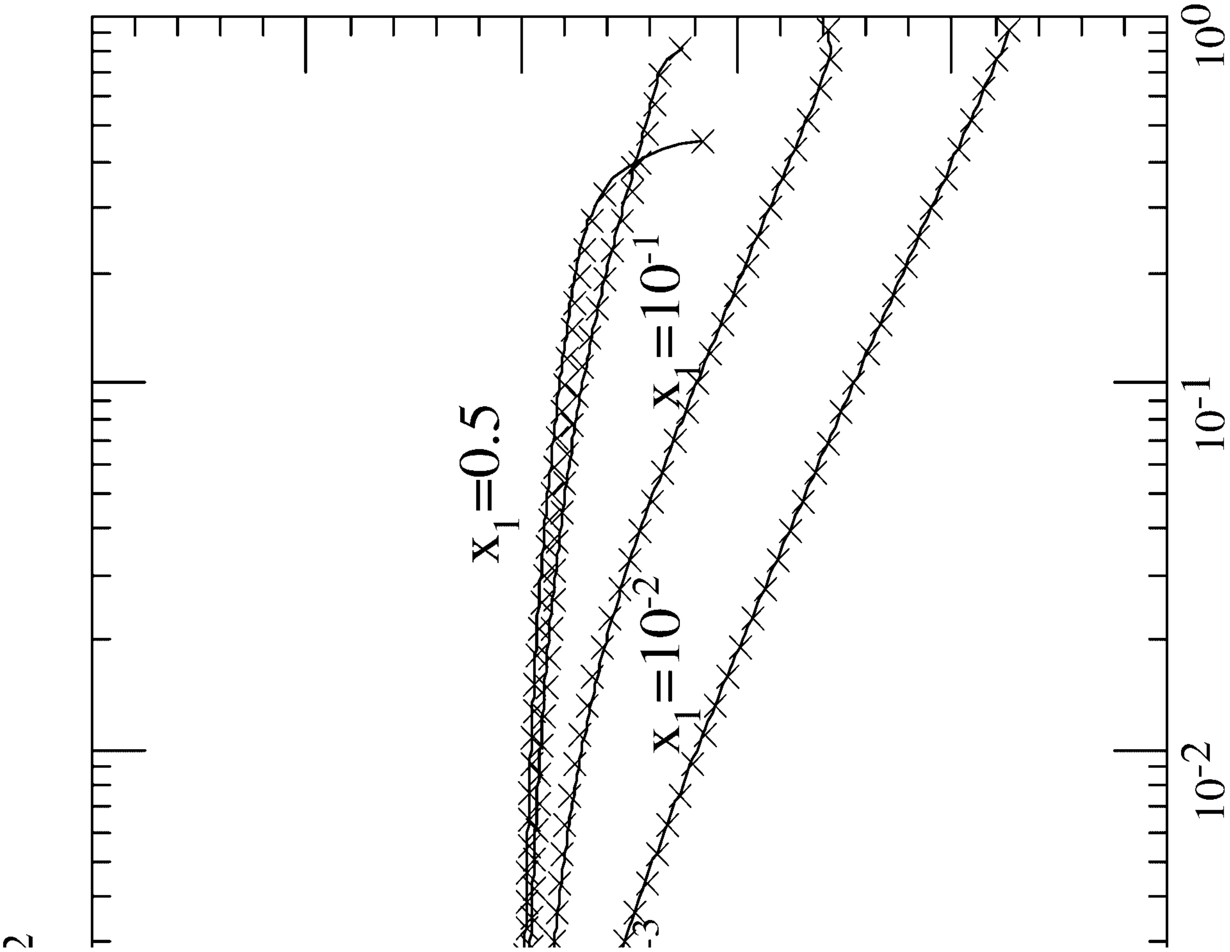

Fig. 3. Fig.2(b)

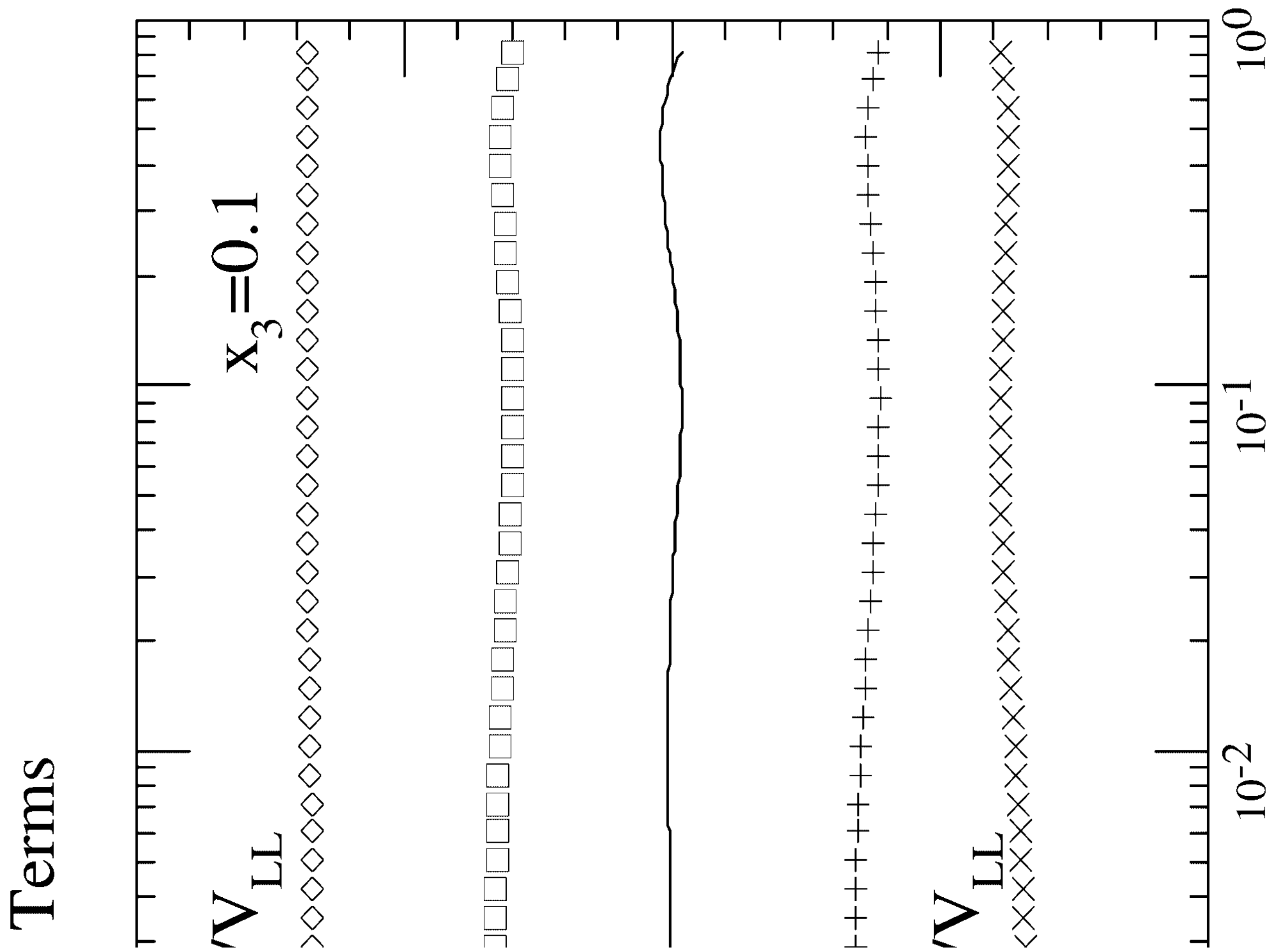

Fig. 4. Fig.3