Note on the Non-Linear Gauge

Kiyoshi KATO
Department of Physics, Kogakuin University
Shinjuku, Tokyo 163-8677, Japan

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1 Tree Lagrangian

The standard model for the electroweak theory is the renormalizable gauge theory whose gauge group is $SU(2) \times U(1)$. We denote the generators of the gauge group as $T^a(a = 0, 1, 2, 3)$ where indices $a = 1, 2, 3$ and $a = 0$ correspond to $SU(2)$ group and $U(1)$ group, respectively. The generator of $U(1)$ group $Y(= T^0)$ is called hypercharge. Since it is Abelian, T_0 commutes all generators. We use the same notation for the generator and its matrix representation. The normalization of T^a is determined by

$$Tr(T^aT^b) = \frac{1}{2}\delta^{ab}. \tag{1}$$

The algebra of generators is

$$[T^a, T^b] = if^{abc}T^c, \tag{2}$$

where f^{abc} are called as structure constants. The generators follow the Jacobi identity as

$$[T^a, [T^b, T^c]] + [T^b, [T^c, T^a]] + [T^c, [T^a, T^b]] = 0, \tag{3}$$

or

$$f^{bcx}f^{axy} + f^{cax}f^{bxy} + f^{abx}f^{cxy} = 0. \tag{4}$$

The structure constants for $SU(2) \times U(1)$ is given by

$$f^{abc} = \begin{cases} \varepsilon^{abc} & (a, b, c \neq 0) \\ 0 & \text{(others)} \end{cases} . \tag{5}$$

Explicit matrix representations for doublets are as follows.

$$T^a = \frac{1}{2}\sigma^a, (a = 1, 2, 3) \qquad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{6}$$

Here σ^a are Pauli matrices.

The group $SU(2) \times U(1)$ breaks down spontaneously as is described later related to Higgs field. After the breakdown, the remaining symmetry is the electro-magnetic symmetry $U_{em}(1)$ whose charge Q is given by

$$Q = I^3 + Y \tag{7}$$

where I^3 is the eigenvalue of T^3. The strength of interaction is determined by the two couplings g and g'.

$$g \cdots SU(2) \qquad g' \cdots U(1) \tag{8}$$

The theory is constructed by the following fields.

1. Gauge fields

   In the classical sense, the gauge vector bosons are the media of electroweak force. We have 4-component gauge fields $W_\mu^a (a = 0, 1, 2, 3)$ for the group $SU(2) \times U(1)$. Later, they become physical $A_\mu, Z_\mu, W_\mu^\pm$ after absorbing longitudinal freedom through Higgs mechanism. At this stage, W_μ^a's are massless and obey the symmetry $SU(2) \times U(1)$. They belongs to the adjoint representation of the gauge group:

$$\delta W_\mu^a = \partial_\mu \theta^a + g f^{abc} W_\mu^b \theta^c \tag{9}$$

   where $\theta^a(x)$ are the c-number functions for the gauge transformation. It should be noted that since $f^{0bc} = 0$ we do not need to use g' for these equations. However, sometimes it is convenient to introduce a notation

$$g^a = \begin{cases} g & (a = 1, 2, 3) \\ g' & (a = 0) \end{cases} . \tag{10}$$

   When we use the notation g^a, the summation over a is often suppressed.

   The field strength $F_{\mu\nu}^a$

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g f^{abc}\ W_\mu^b W_\nu^c \tag{11}$$

   is covariant by the transformation Eq.9. We sometimes use the short-hand notation

$$\partial_{[\mu} W_{\nu]} = \partial_\mu W_\nu - \partial_\nu W_\mu . \tag{12}$$

   The relation

$$\delta F_{\mu\nu}^a = g f^{abc} F_{\mu\nu}^b \theta^c . \tag{13}$$

   holds with the help of Jacobi identity. Since $\delta(F_{\mu\nu}^a F^{a\mu\nu}) = 2\delta F_{\mu\nu}^a F^{a\mu\nu} = 0$, we can define the gauge invariant Lagrangian for the gauge fields:

$$L(gauge) = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \tag{14}$$

   We separate this Lagrangian by the order of fields as

$$L(gauge) = L^{(2)}(gauge) + L^{(3)}(gauge) + L^{(4)}(gauge). \tag{15}$$

   The bilinear term is

$$L^{(2)}(gauge) = -\frac{1}{4} \partial_{[\mu} W_{\nu]}^a \partial^{[\mu} W^{a\nu]} = \frac{1}{2} W_\mu^a \left(\partial_\alpha \partial^\alpha g^{\mu\nu} - \partial^\mu \partial^\nu \right) W_\nu^a . \tag{16}$$

   As the operator $\partial_\alpha \partial^\alpha g^{\mu\nu} - \partial^\mu \partial^\nu$ has no inverse, the covariant propagator cannot be defined. The gauge fixing resolves this point.

2. Matter fields

The matter fields are fermions and scalars. They belong to the fundamental representation of the gauge group. If we write a matter field as $\phi(x)$, its gauge transformation is given by

$$\delta\phi = i\sum_a g^a\theta^a T^a\phi. \tag{17}$$

Below, we drop the summation symbol for a. (Of course, a pair of an index implicitly means the summation over the index. However, here three a's appear, so that the summation is shown explicitly.) The covariant derivative of a matter field is defined by

$$D_\mu\phi = \partial_\mu\phi - ig^a W_\mu^a T^a\phi \tag{18}$$

and

$$\delta(D_\mu\phi) = i\theta^a g^a T^a D_\mu\phi. \tag{19}$$

Then the gauge invariant matter Lagrangian can be the combination of the following terms.

$$\bar\psi i\gamma^\mu D_\mu\psi, \quad (D_\mu\phi)^\dagger(D^\mu\phi), \quad \phi^\dagger\phi. \tag{20}$$

By Eq.6, the explicit form of the covariant derivative is

$$SU(2)\text{ doublet}\quad D_\mu = \begin{pmatrix} \partial_\mu - igW_\mu^3/2 - ig'YW_\mu^0 & -ig(W_\mu^1 - iW_\mu^2)/2 \\ -ig(W_\mu^1 + iW_\mu^2)/2 & \partial_\mu + igW_\mu^3/2 - ig'YW_\mu^0 \end{pmatrix}, \tag{21}$$

$$SU(2)\text{ singlet}\qquad D_\mu = \partial_\mu - ig'YW_\mu^0. \tag{22}$$

Now we explicitly specify the matter fields with their quantum numbers. In the table, the generic names (e.g., f, q_L and so on) are also shown. Though it is not explicitly shown, the quarks have color degree of freedom.

fermion(f)			I^3	Y	Q
left-handed fermion (f_L)					
quark (q_L)	$\cdots\begin{pmatrix} U_L \\ D_L \end{pmatrix}$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
lepton (l_L)	$\cdots\begin{pmatrix} U_L \\ D_L \end{pmatrix}$	$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	-$\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
right-handed fermion (f_R)					
quark (q_R)	$\cdots U_R$	u_R, c_R, t_R	0	$\frac{2}{3}$	$\frac{2}{3}$
	$\cdots D_R$	d_R, s_R, b_R	0	$-\frac{1}{3}$	$-\frac{1}{3}$
lepton (l_R)	$\cdots U_R$	e_R, μ_R, τ_R	0	-1	-1
scalar(ϕ)			I^3	Y	Q
(See next section.)		$\frac{1}{\sqrt{2}}\begin{pmatrix} i\chi_1 + \chi_2 \\ v + H - i\chi_3 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

The suffixes L and R represent the left- and right-handed components, respectively.

$$\psi_L = L\psi = \frac{1}{2}(1-\gamma_5)\psi, \quad \psi_R = R\psi = \frac{1}{2}(1+\gamma_5)\psi. \tag{23}$$

The "bar" of ψ is defined as usual

$$\bar{\psi} = \psi^\dagger \gamma^0 \tag{24}$$

so that

$$\bar{\psi_L} = \bar{\psi} R,\ \bar{\psi_R} = \bar{\psi} L \quad \Rightarrow \quad \bar{\psi}_{1L}\psi_{2L} = \bar{\psi}_{1R}\psi_{2R} = 0. \tag{25}$$

The mass term for fermions should appear as $\sim m\bar{\psi}_L\psi_R + (h.c.)$. However, as is shown in the table, it is impossible to write the mass term explicitly without breaking gauge invariance. We can make the combination $\sim \bar{f}_L \phi f_R$ and the fermion mass terms are generated from the vacuum expectation value of ϕ. Then the coupling of fermion and scalar turns to be proportional to the mass of fermion. This is explicitly shown in Sec.2.2.

The matter Lagrangian becomes as follows:

$$L(fermion) + L(salar) \tag{26}$$

Fermionic part is

$$L(fermion) = \sum \bar{f}_L i\gamma^\mu D_\mu f_L + \sum \bar{f}_R i\gamma^\mu D_\mu f_R = L(f, kin) + L(g - f) \tag{27}$$

where the sum is for the all left-handed doublets and all right-handed singlets. Scalar part is

$$L(scalar) = L(s, kin) + L(s - f) + L(pot) \tag{28}$$

where

$$L(s, kin) = (D_\mu\phi)^\dagger(D^\mu\phi), \tag{29}$$

$$L(s - f) = -\sum f_U \bar{f}_L \tilde{\phi} U_R - \sum f_D \bar{f}_L \phi D_R + (h.c.), \tag{30}$$

$$L(pot) = -V(\phi) = \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2. \tag{31}$$

The last term, $L(pot)$ is related the spontaneous symmetry breakdown as is discussed in Sec.2.3. In the scalar-fermion interaction term, we use the notation

$$\tilde{\phi} = i\sigma^2\phi^* \tag{32}$$

under the representation in Eq.6. The $\tilde{\phi}$ transforms as follows. When we take the complex conjugate of Eq.17,

$$\delta\phi^* = -i\sum_{a=1,2,3} g\theta^a \frac{1}{2}(\sigma^a)^*\phi^* - ig'\theta^0 T^0\phi^*. \tag{33}$$

Since for Pauli matrices the relation

$$(\sigma^2)^{-1}\sigma^a(\sigma^2) = -(\sigma^a)^* \tag{34}$$

holds,

$$\delta\tilde{\phi} = i\sum_{a=1,2,3} g\theta^a T^a\tilde{\phi} - ig'\theta^0 T^0\tilde{\phi}. \tag{35}$$

Thus $\tilde{\phi}$ has the same $SU(2)$ transformation property as ϕ and its hypercharge is $-Y$.

3. Ghost fields

Through the gauge fixing, we are to introduce auxiliary field and ghost fields. This is discussed in Sec.3.

2 Higgs sector

In this section we study terms of $L(scalar)$ in Eq.28 in detail. The parts $L(s, kin)$, $L(s-f)$, and $L(pot)$, are discussed in Sec.2.1, Sec.2.2, and Sec.2.3, respectively.

2.1 Higgs mechanism

The potential part of scalar, Eq.31, has non trivial minimum since it has a negative coefficient in quadratic term. The minimum is the vacuum expectation value of the field ϕ. The fields used in the perturbation are to be defined as the fluctuation around the minimum.

As given in the last section, the scalar field ϕ is defined as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\chi_1 + \chi_2 \\ v + H - i\chi_3 \end{pmatrix} = \begin{pmatrix} i\chi^+ \\ (v + H - i\chi_3)/\sqrt{2} \end{pmatrix} \tag{36}$$

where $\chi^\pm = (\chi_1 \mp i\chi_2)/\sqrt{2}$. We have introduced v in order to represent the non-trivial vacuum expectation value as in Sec.2.3. At this stage, v is just a parameter in the theory, and as will be discussed below, v is determined by three physical parameters, e, M_W, M_Z.(Eq.47)

From the definition of the electric charge($Q = T^3 + Y$, Eq.7) and the covariant derivative in Eq.21 and Eq.22, the photon field A_μ should be contained in the neutral gauge bosons as $W^3 \sim Cg'A$ and $W^0 \sim CgA$. Another physical neutral gauge boson Z_μ is defined orthogonal to A_μ. Also, we define the charged boson states.

$$\begin{cases} A_\mu = \dfrac{g'W^3_\mu + gW^0_\mu}{\sqrt{g^2 + g'^2}} = s_W W^3_\mu + c_W W^0_\mu \\[2ex] Z_\mu = \dfrac{gW^3_\mu - g'W^0_\mu}{\sqrt{g^2 + g'^2}} = c_W W^3_\mu - s_W W^0_\mu \\[2ex] W^\pm_\mu = \dfrac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}} \end{cases} \tag{37}$$

where the notation

$$c_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad s_W = \frac{g'}{\sqrt{g^2 + g'^2}} \tag{38}$$

is used. The reversed relations are

$$\begin{cases} W^0_\mu = c_W A_\mu - s_W Z_\mu \\ W^3_\mu = s_W A_\mu + c_W Z_\mu \\ W^1_\mu = (W^+_\mu + W^-_\mu)/\sqrt{2} \\ W^2_\mu = i(W^+_\mu - W^-_\mu)/\sqrt{2} \end{cases} \tag{39}$$

By use of these definitions, Eq.21 and Eq.22 become as below.

$$SU(2) \text{ doublet} \quad D_\mu =$$

$$\begin{pmatrix} \partial_\mu - ieQA_\mu - ie(I^3 - Qs^2_W)/s_W c_W Z_\mu & -ieW^+_\mu/\sqrt{2}s_W \\ -ieW^-_\mu/\sqrt{2}s_W & \partial_\mu - ieQA_\mu - ie(I^3 - Qs^2_W)/s_W c_W Z_\mu \end{pmatrix}, \tag{40}$$

$$SU(2) \text{ singlet} \qquad D_\mu = \partial_\mu - ieQA_\mu + ieQ\frac{s_W}{c_W}Z_\mu \tag{41}$$

where

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad g = \frac{e}{s_W}, \quad g' = \frac{e}{c_W}. \tag{42}$$

As in the table, the charge of ϕ field is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We explicitly write down $L(s, kin)$ substituting Eq.36 and Eq.40.

$$D^\mu \phi =$$

$$\begin{pmatrix} i\left(\partial_\mu \chi^+ - \frac{ev}{2s_W} W^+_\mu\right) + e\left(-i\frac{1}{2s_W} W^+_\mu H - \frac{1}{2s_W} W^+_\mu \chi_3 + A_\mu \chi^+ + \frac{1-2s_W^2}{2s_W c_W} Z_\mu \chi^+\right) \\ \frac{1}{\sqrt{2}}\left(\partial_\mu H - i\left(\partial_\mu \chi_3 - \frac{ev}{2s_W c_W} Z_\mu\right)\right) + \frac{e}{\sqrt{2}s_W}\left(W^-_\mu \chi^+ + \frac{1}{2c_W} Z_\mu \chi_3 + i\frac{1}{2c_W} Z_\mu H\right) \end{pmatrix} \tag{43}$$

We separate this Lagrangian by the order of fields as

$$L(s, kin) = (D_\mu \phi)^\dagger D^\mu \phi = L^{(2)}(s, kin) + L^{(3)}(s, kin) + L^{(4)}(s, kin). \tag{44}$$

The bilinear part is as follows:

$$L^{(2)}(s, kin) = (\partial_\mu \chi^- - M_W W^-_\mu)(\partial^\mu \chi^+ - M_W W^{+\mu}) + \frac{1}{2}(\partial_\mu \chi_3 - M_Z Z_\mu)^2 + \frac{1}{2}(\partial_\mu H)^2 \tag{45}$$

where

$$M_W = \frac{ev}{2s_W}, \qquad M_Z = \frac{ev}{2s_W c_W}. \tag{46}$$

This term provides mass terms for gauge bosons $Z, W^\pm$. By Eq.46, the parameter v is given by

$$v = \frac{2s_W M_W}{e} = \frac{2\sqrt{M_Z^2 - M_W^2} M_W}{e M_Z} \tag{47}$$

If we redefine the fields as

$$\tilde{W}^\pm_\mu = W^\pm_\mu - \frac{1}{M_W}\partial_\mu \chi^\pm, \quad \tilde{Z}_\mu = Z_\mu - \frac{1}{M_Z}\partial_\mu \chi_3, \tag{48}$$

the fields of unphysical particles, χ's, becomes just auxiliary fields. This is the Higgs mechanism, i.e., massless Goldstone bosons are absorbed as longitudinal part of vector fields.

We do not use this redefinitions here since the gauge fixing is not yet done. The discussion continues to Sec.3.4.

The rest of the Lagrangian is as follows.

$$\begin{aligned} L^{(3)}(s, kin) = \quad & -\frac{e}{2s_W}(\partial_\mu \chi^- - M_W W^-_\mu) W^{+\mu} H + h.c. \\ & -i\frac{e}{2s_W}(\partial_\mu \chi^- - M_W W^-_\mu)\left(-\frac{1}{2s_W} W^{+\mu}\chi_3 + A^\mu \chi^+ + \frac{1-2s_W^2}{2s_W c_W} Z_\mu \chi^+\right) + h.c. \\ & +\frac{e}{2s_W}\partial_\mu H\left(W^{+\mu}\chi^- + W^{-\mu}\chi^+ + \frac{1}{c_W} Z^\mu \chi_3\right) \\ & -\frac{e}{2s_W c_W}(\partial_\mu \chi_3 - M_Z Z_\mu) Z^\mu H + i\frac{e}{2s_W}(\partial_\mu \chi_3 - M_Z Z_\mu)(W^{-\mu}\chi^+ - W^{+\mu}\chi^-) \end{aligned} \tag{49}$$

This can be written as

$$L^{(3)}(s,kin) = L^{(3,SSV)}(s,kin) + L^{(3,VVS)}(s,kin) \tag{50}$$

where

$$\begin{aligned} L^{(3,SSV)}(s,kin) = \; & ieA_\mu(\chi^- \overset{\leftrightarrow}{\partial^\mu} \chi^+) + i\frac{e(1-2s_W^2)}{2s_W c_W} Z_\mu(\chi^- \overset{\leftrightarrow}{\partial^\mu} \chi^+) + \frac{e}{2s_W c_W} Z_\mu(\chi_3 \overset{\leftrightarrow}{\partial^\mu} H) \\ & + \frac{e}{2s_W}\left(W_\mu^+(\chi^- \overset{\leftrightarrow}{\partial^\mu} H) + W_\mu^-(\chi^+ \overset{\leftrightarrow}{\partial^\mu} H)\right) \\ & + i\frac{e}{2s_W}\left(-W_\mu^+(\chi^- \overset{\leftrightarrow}{\partial^\mu} \chi_3) + W_\mu^-(\chi^+ \overset{\leftrightarrow}{\partial^\mu} \chi_3)\right), \end{aligned} \tag{51}$$

and

$$\begin{aligned} L^{(3,VVS)}(s,kin) = \; & \frac{e}{s_W} M_W W_\mu^- W^{+\mu} H + \frac{e}{2s_W c_W} M_Z Z_\mu Z^\mu H \\ & + ie(M_W A_\mu - s_W M_Z Z_\mu)(W^{-\mu}\chi^+ - W^{+\mu}\chi^-). \end{aligned} \tag{52}$$

Here the notation

$$a \overset{\leftrightarrow}{\partial^\mu} b = a(\partial^\mu b) - (\partial^\mu a) b \tag{53}$$

is used.

$$\begin{aligned} L^{(4)}(s,kin) = \; & e^2\left(+i\frac{1}{2s_W} W_\mu^- H - \frac{1}{2s_W} W_\mu^- \chi_3 + A_\mu \chi^- + \frac{1-2s_W^2}{2s_W c_W} Z_\mu \chi^-\right) \\ & \times \left(-i\frac{1}{2s_W} W^{+\mu} H - \frac{1}{2s_W} W^{+\mu}\chi_3 + A^\mu \chi^+ + \frac{1-2s_W^2}{2s_W c_W} Z^\mu \chi^+\right) \\ & + \frac{e^2}{2s_W^2}\left(W_\mu^+ \chi^- + \frac{1}{2c_W} Z_\mu \chi_3 - i\frac{1}{2c_W} Z_\mu H\right) \\ & \times \left(W^{-\mu}\chi^+ + \frac{1}{2c_W} Z_\mu \chi_3 + i\frac{1}{2c_W} Z_\mu H\right). \end{aligned} \tag{54}$$

This can be written as

$$\begin{aligned} L^{(4)}(s,kin) = \; & e^2 A_\mu A^\mu \chi^+ \chi^- + \frac{e^2(1-2s_W^2)}{s_W c_W} A_\mu Z^\mu \chi^+ \chi^- \\ & + \frac{e^2}{8s_W^2 c_W^2} Z_\mu Z^\mu \left(2(1-2s_W^2)^2 \chi^+\chi^- + (H^2 + \chi_3^2)\right) \\ & + \frac{e^2}{4s_W^2} W_\mu^- W^{+\mu}(H^2 + \chi_3^2 + 2\chi^+\chi^-) \\ & - \frac{e^2}{2s_W}\left[i(W_\mu^+ A^\mu \chi^- - W_\mu^- A^\mu \chi^+)H + (W_\mu^+ A^\mu \chi^- + W_\mu^- A^\mu \chi^+)\chi_3\right] \\ & + \frac{e^2}{2c_W}\left[i(W_\mu^+ Z^\mu \chi^- - W_\mu^- Z^\mu \chi^+)H + (W_\mu^+ Z^\mu \chi^- + W_\mu^- Z^\mu \chi^+)\chi_3\right]. \end{aligned} \tag{55}$$

2.2 Interaction with fermion

The interaction term between fermion and scalar is as follows.(Eq.30)

$$\begin{aligned} L(s-f) = \quad & -\sum f_U\,(\bar{U}_L\ \bar{D}_L)\begin{pmatrix} \frac{1}{\sqrt{2}}(v+H+i\chi_3) \\ i\chi^- \end{pmatrix} U_R + (h.c.) \\ & -\sum f_D\,(\bar{U}_L\ \bar{D}_L)\begin{pmatrix} i\chi^+ \\ \frac{1}{\sqrt{2}}(v+H-i\chi_3) \end{pmatrix} D_R + (h.c.). \end{aligned} \tag{56}$$

We separate the mass terms and interaction terms.

$$L(s-f) = L^{(2)}(s-f) + L^{(3)}(s-f) \tag{57}$$

$$L^{(2)}(s-f) = -\sum m_f \bar{f} f \tag{58}$$

where

$$m_{U,D} = \frac{f_{U,D}\,v}{\sqrt{2}}. \tag{59}$$

$$\begin{aligned} L^{(3)}(s-f) = \quad & -\sum \frac{m_f}{v}\bar{f} f H \\ & -\sum i\frac{m_U}{v}\bar{U}\gamma_5 U\chi_3 + \sum i\frac{m_D}{v}\bar{D}\gamma_5 D\chi_3 \\ & -\sum i\frac{\sqrt{2}m_U}{v}\bar{D}RU\chi^- + \sum i\frac{\sqrt{2}m_D}{v}\bar{D}LU\chi^- \\ & -\sum i\frac{\sqrt{2}m_D}{v}\bar{U}RD\chi^+ + \sum i\frac{\sqrt{2}m_U}{v}\bar{U}LD\chi^+ \end{aligned} \tag{60}$$

Here the relation(Eq.47)

$$\frac{1}{v} = \frac{e}{2s_W M_W} \tag{61}$$

is to be used.

2.3 Scalar potential

The field ϕ is parameterized by Eq.36. Then

$$\phi^\dagger\phi = \frac{v^2}{2} + vH + \frac{1}{2}H^2 + \frac{1}{2}\chi_3^2 + \chi^+\chi^- \tag{62}$$

and the potential part in Eq.31 is given as follows:

$$L(pot) = const. + v(\mu^2 - \lambda v^2)H + L^{(2)}(pot) + L^{(3)}(pot) + L^{(4)}(pot) \tag{63}$$

where

$$L^{(2)}(pot) = \frac{1}{2}(\mu^2 - 3\lambda v^2)H^2 + \frac{1}{2}(\mu^2 - \lambda v^2)\chi_3^2 + (\mu^2 - \lambda v^2)\chi^+\chi^-, \tag{64}$$

$$L^{(3)}(pot) = -\lambda v(H^3 + H\chi_3^2 + 2H\chi^+\chi^-), \tag{65}$$

$$L^{(4)}(pot) = -\lambda\left(\frac{H^4}{4} + \frac{\chi_3^4}{4} + (\chi^+\chi^-)^2 + \frac{H^2\chi_3^2}{2} + H^2\chi^+\chi^- + \chi_3^2\chi^+\chi^-\right). \tag{66}$$

Here, we introduce two new notations instead of μ^2 and λ. (v is defined in Sec.2.1.)

$$T = v(\mu^2 - \lambda v^2), \tag{67}$$

$$M_H^2 = 2\mu^2. \tag{68}$$

By these notations $L(pot)$ becomes as follows:

$$L(pot) = const. + TH + L^{(2)}(pot) + L^{(3)}(pot) + L^{(4)}(pot) \tag{69}$$

where

$$L^{(2)}(pot) = -\frac{1}{2}\left(M_H^2 - \frac{3T}{v}\right)H^2 + \frac{1}{2}\frac{T}{v}\chi_3^2 + \frac{T}{v}\chi^+\chi^-, \tag{70}$$

$$L^{(3)}(pot) = \left(\frac{T}{v^2} - \frac{eM_H^2}{4s_W M_W}\right)(H^3 + H\chi_3^2 + 2H\chi^+\chi^-), \tag{71}$$

$$L^{(4)}(pot) = \left(\frac{T}{v^3} - \frac{e^2 M_H^2}{8s_W^2 M_W^2}\right)\left(\frac{H^4}{4} + \frac{\chi_3^4}{4} + (\chi^+\chi^-)^2 + \frac{H^2\chi_3^2}{2} + H^2\chi^+\chi^- + \chi_3^2\chi^+\chi^-\right). \tag{72}$$

In the tree level, if we require the condition that v specify the minimum of the potential, we obtain $T = 0$. So we can set $T = 0$ for the definition of tree Feynman rules. For the tadpole renormalization, we keep this notation. The terms in $L^{(2)}(pot)$ corresponds to the masses of scalar particles. In the tree level, the physical Higgs particle H acquires the mass M_H and the masses of χ's are 0, i.e., they are Goldstone bosons related to the broken symmetry. The terms in $L^{(3)}(pot)$ and $L^{(4)}(pot)$ are the interaction terms between scalars.

Now we check that $T = 0$ determines the potential minimum. The potential part of scalar, Eq.31, can be written as follows.

$$V(\phi) = \lambda\left((\phi^\dagger\phi) - \frac{\mu^2}{2\lambda}\right)^2 - \frac{\mu^4}{4\lambda} \tag{73}$$

and we define

$$<\phi^\dagger\phi> = \frac{v^2}{2} = \frac{\mu^2}{2\lambda}. \tag{74}$$

Thus $T = 0$ corresponds to the minimum of tree potential.

2.4 $L(gauge)$ and $L(fermion)$ in physical fields

In Sec.1, the Lagrangian for gauge field is defined by Eq.14 and that for fermion is done by Eq.27. We have defined physical fields by Eq.39. In this subsection, we write $L(gauge)$ and $L(fermion)$ by these fields.

1. $L(gauge)$

$$\left\{\begin{array}{rcl} F^0_{\mu\nu} &=& c_W\partial_{[\mu}A_{\nu]} - s_W\partial_{[\mu}Z_{\nu]} \\ F^3_{\mu\nu} &=& s_W\partial_{[\mu}A_{\nu]} + c_W\partial_{[\mu}Z_{\nu]}ig(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu) \\ F^1_{\mu\nu} &=& \frac{1}{\sqrt{2}}(\partial_{[\mu}W^+_{\nu]} + \partial_{[\mu}W^-_{\nu]}) \\ && +\frac{ie}{\sqrt{2}s_W}[(W^+_\mu - W^-_\mu)(s_W A^\nu + c_S Z^\nu) - (W^+_\nu - W^-_\nu)(s_W A^\mu + c_S Z^\mu)] \\ F^2_{\mu\nu} &=& \frac{i}{\sqrt{2}}(\partial_{[\mu}W^+_{\nu]} - \partial_{[\mu}W^-_{\nu]}) \\ && +\frac{e}{\sqrt{2}s_W}[-(W^+_\mu + W^-_\mu)(s_W A^\nu + c_S Z^\nu) + (W^+_\nu + W^-_\nu)(s_W A^\mu + c_S Z^\mu)] \end{array}\right. \tag{75}$$

The we obtain the followings:

$$L^{(2)}(gauge) = -\frac{1}{4}\partial_{[\mu}A_{\nu]}\partial^{[\mu}A^{\nu]} - \frac{1}{4}\partial_{[\mu}Z_{\nu]}\partial^{[\mu}Z^{\nu]} - \frac{1}{2}\partial_{[\mu}W^{-}_{\nu]}\partial^{[\mu}W^{+\nu]} \tag{76}$$

$$\begin{aligned} L^{(3)}(gauge) = \;& -\left(\frac{ie}{2}\partial_{[\mu}A_{\nu]} + \frac{iec_W}{2s_W}\partial_{[\mu}Z_{\nu]}\right)(W^{-\mu}W^{+\nu} - W^{-\nu}W^{+\mu}) \\ & + \frac{ie}{2s_W}\left(\partial_{[\mu}W^{+}_{\nu]}W^{-\mu} - \partial_{[\mu}W^{-}_{\nu]}W^{+\mu}\right)(s_W A^\nu + c_W Z^\nu) \end{aligned} \tag{77}$$

$$\begin{aligned} L^{(4)}(gauge) = \;& +\frac{e^2}{4s_W^2}(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)(W^{-\mu}W^{+\nu} - W^{-\nu}W^{+\mu}) \\ & -\frac{e^2}{2s_W^2}W^-_\mu W^{+\mu}(s_W A_\nu + c_W Z_\nu)(s_W A^\nu + c_W Z^\nu) \\ & +\frac{e^2}{4s_W^2}(W^-_\mu W^+_\nu + W^-_\nu W^+_\mu)(s_W A^\mu + c_W Z^\mu)(s_W A^\nu + c_W Z^\nu) \end{aligned} \tag{78}$$

2. $L(fermion)$

$$\begin{aligned} L(fermion) &= L(f,kin) + L(g-f) \\ &= \sum_{(U,D)} \begin{pmatrix} \bar{U}_L \\ \bar{D}_L \end{pmatrix} i\gamma^\mu D_\mu (U_L \; D_L) + \sum_U \bar{U}_R i\gamma^\mu D_\mu U_R + \sum_D \bar{D}_R i\gamma^\mu D_\mu D_R \end{aligned} \tag{79}$$

and derivative D_μ is given in Eq.40 and Eq.41.

$$L(f,kin) = \sum_f \bar{f} i\gamma^\mu \partial_\mu f \tag{80}$$

$$\begin{aligned} L(f, g-f) = \;& \frac{e}{\sqrt{2}s_W}\sum_{(U,D)} \bar{U}\gamma^\mu L D W^+_\mu + \bar{D}\gamma^\mu L U W^-_\mu \\ & + e\sum_f Q\bar{f}\gamma^\mu f A_\mu \\ & + \frac{e}{2s_W c_W}\sum_f \bar{f}\gamma^\mu[2I_3 L - 2Qs_W^2] f Z_\mu \end{aligned} \tag{81}$$

3 Gauge fixing

3.1 Auxiliary fields

Based on the gauge principle, the Lagrangian is constructed so that it is invariant under the gauge transformation. However, this means that the gauge fields have redundant freedom. In order to construct quantum theory of gauge fields, we must choose only the independent modes. Thus the gauge fixing condition is introduced.

Here, we implicitly assume the quantization by the path integral method. The gauge fixing condition can be written as

$$F^a[W] = f^a(x) \tag{82}$$

where $f(x)$'s are a set of arbitrary function. Here, $F[W]$ is a function of gauge fields to fix the gauge. For an instance, $F^a[W] = \partial^\mu W^a_\mu$ determines so-called covariant gauge. To use this condition, we insert hyper-product of delta functions

$$\prod_{x,a} \delta\left(F^a[W] - f^a(x)\right) \tag{83}$$

into the path-integral formula for the transition amplitude.

The physical result should be independent of the explicit form of $f(x)$. The role of $f(x)$ is to choose one representative point from a gauge trajectory and each point on a trajectory is equivalent to each other. So it makes no change when we average over $f(x)$. We consider the following integral

$$\int \mathcal{D}f \exp\left[i \int d^4x \left(-\frac{1}{2\xi} f^a(x) f^a(x)\right)\right] = (const.) \tag{84}$$

and since it is a constant we can multiply it to the transition amplitude. Here ξ is an arbitrary number and called as the gauge parameter.

We introduce another variable $B^a(x)$ and convert the above constant.

$$\int \mathcal{D}B\mathcal{D}f \exp\left[i \int d^4x \left(\frac{\xi}{2} B^a(x) B^a(x) + B^a(x) f^a(x)\right)\right] = (const.) \tag{85}$$

When we integrate over $B^a(x)$, Eq.85 becomes Eq.84. (Overall infinite number has no sense as usual.)

We multiply the gauge fixing condition, Eq.83, the constant number given by Eq.85, and integrate over $f^a(x)$ to get

$$\int \mathcal{D}B \exp\left[i \int d^4x \left(\frac{\xi}{2} B^a(x) B^a(x) + B^a(x) F^a[W]\right)\right]. \tag{86}$$

If we integrate over $B^a(x)$, Eq.86 turns to

$$\exp\left[i \int d^4x \left(-\frac{1}{2\xi} (F^a[W])^2\right)\right]. \tag{87}$$

We can conclude as follows. The gauge fixing Eq.83 is equivalent to either of the following procedures.

1. Add a term

   $$-\frac{1}{2\xi} (F^a[W])^2 \tag{88}$$

   into the original Lagrangian.

2. Introduce a new field B and add terms

   $$\frac{\xi}{2} B^a B^a + B^a F^a[W] \tag{89}$$

   into the original Lagrangian. The field B^a is called as auxiliary fields since they have no kinetic terms like $(\partial_\mu B)^2$.

   The Eq.89 for charged fields is as follows:

   $$\xi B^+ B^- + B^+ F^- + B^- F^+ \tag{90}$$

Then $B^{\pm} = (B^1 \mp iB^2)/\sqrt{2}$ gives

$$\frac{\xi}{2}(B^1B^1 + B^2B^2) + B^1(F^+ + F^-)/\sqrt{2} + iB^2(F^+ - F^-)/\sqrt{2}$$
$$= \frac{\xi}{2}(B^1B^1 + B^2B^2) + B^1F^1 + B^2F^2 \tag{91}$$

to give Eq.88 after integration.

3.2 BRS symmetry

The introduction of the gauge fixing term given in the last subsection leads another counter action. The systematic treatment can be done through the idea of a new symmetry principle. In Sec.1, we already defined the gauge symmetry of the theory in classical level. The BRS symmetry can be considered as the symmetry for the quantum theory. The use of BRS symmetry provides us the clear understanding about the structure of gauge theory, including unitarity, renormalizability and so forth. We construct the quantized electroweak theory by the formulation based on the BRS symmetry.

The transformation of fields by the BRS symmetry is obtained by the replacement

$$\theta^a(x) \to \Lambda c^a(x) \tag{92}$$

in the classical transformation in Sec.1. Here, both Λ and $c^a(x)$ are Grassmann variables and the latter $c^a(x)$ is called as the ghost field. Corresponding to Eq.9 and Eq.17, the BRS variation for gauge fields and matter fields is given by

$$\Delta W^a_\mu = \Lambda(\partial_\mu c^a + gf^{abc}W^b_\mu c^c) \tag{93}$$

and

$$\Delta\phi = \Lambda(i\sum_a g^a c^a T^a \phi). \tag{94}$$

We introduce a modified BRS variation δ_B as

$$\Delta(\cdots) = \Lambda\delta_B(\cdots). \tag{95}$$

Since we have separated Λ, δ_B itself has Grassmann nature, i.e.,

$$\delta_B(AB) = \begin{cases} \delta_B(A)B + A\delta_B(B) & (A \textit{ not } \text{Grassmann}) \\ \delta_B(A)B - A\delta_B(B) & (A \text{ Grassmann}) \end{cases}. \tag{96}$$

In the last subsection we have introduced the auxiliary fields. We introduce the anti-ghost field, $\bar{c}^a$, so that its BRS variation gives B field. The BRS variation of fields is summarized as follows:

$$\delta_B W^a_\mu = \partial_\mu c^a + gf^{abc}W^b_\mu c^c \tag{97}$$
$$\delta_B\phi = i\sum_a g^a c^a T^a \phi \tag{98}$$
$$\delta_B c^a = -\frac{1}{2}gf^{abc}c^b c^c \tag{99}$$
$$\delta_B \bar{c}^a = B^a \tag{100}$$
$$\delta_B B^a = 0 \tag{101}$$

The BRS variation δ_B is nilpotent, i.e.,

$$\delta_B^2(\cdots) = 0. \tag{102}$$

The variation of the fields above satisfies the nilpotency. As a matter of fact, $\delta_B c^a$ is fixed by the nilpotency as shown below. For the matter fields,

$$\delta_B(\delta_B\phi) = ig^a[\delta_B(c^a)T^a\phi - c^aT^aig^ac^bT^b\phi] = 0 \tag{103}$$

is realized if

$$\delta_B(c^a)T^a = ig^ac^ac^bT^aT^b = g^a\frac{i}{2}c^ac^b[T^a,T^b] = -g^a\frac{1}{2}f^{abc}c^ac^bT^c \tag{104}$$

where we have used that c^a is Grassmann. This proves Eq.99. The check of nilpotency for other fields are as follows: For Eq.97,

$$\delta_B^2W_\mu^a = \partial_\mu(-\frac{1}{2}gf^{abc}c^bc^c) + gf^{abc}[(\partial_\mu c^b + gf^{bxy}W_\mu^xc^y)c^c + W_\mu^b(-\frac{1}{2}gf^{cxy}c^xc^y)] = 0 \tag{105}$$

since

$$-\frac{1}{2}f^{abc}f^{cxy}c^xc^y = -\frac{1}{2}(+f^{acx}f^{cyb} + f^{acy}f^{cbx})c^xc^y = +f^{acx}f^{cby}c^xc^y \tag{106}$$

and for Eq.99

$$\delta_B^2c^a = -g\frac{1}{2}f^{abc}(-g\frac{1}{2}f^{bxy}c^xc^yc^c + c^bg\frac{1}{2}f^{cxy}c^xc^y) = +g^2\frac{1}{2}g^2f^{abc}f^{bxy}c^xc^yc^c = 0 \tag{107}$$

since

$$f^{abc}f^{bxy}c^xc^yc^c = \frac{1}{3}(f^{abc}f^{bxy}c^xc^yc^c + f^{abc}f^{bxy}c^yc^cc^x + f^{abc}f^{bxy}c^cc^xc^y) = 0 \tag{108}$$

Here, the Jacobi identity in Eq.4 is used. $\delta_B^2\overline{c}^a = 0$ and $\delta_B^2B^a = 0$ are trivial.

Here, we have shown that each field component is nilpotent(Eq.102). Then if A and B are both nilpotent ($\delta_B^2A = \delta_B^2B = 0$), the product AB is also nilpotent.

$$\begin{aligned}\delta_B^2(AB) &= \delta_B\left[\delta_B(A)B + \sigma A\delta_B(B)\right] \\ &= \sigma'\delta_B(A)\delta_B(B) + \sigma\delta_B(A)\delta_B(B)\end{aligned} \tag{109}$$

where σ and σ' are signature factors defined in Eq.96, σ for A and σ' for δ_BA. Since the Grassmann property of A differs from that of δ_BA, $\sigma' = -\sigma$, so that $\delta_B^2(AB) = 0$. This proves that any function constructed by nilpotent fields is nilpotent.

For the later use, we write Eq.97 and Eq.98 in physical fields defined in Sec.2.1 and Sec.2.3. The definition of charged ghost fields is $c^\pm = (c^1 \mp ic^2)/\sqrt{2}$ and c^A, c^Z are defined as the same way as A_μ, Z_μ in Eq.39.

$$\delta_BW_\mu^\pm = \partial_\mu c^\pm \pm ie\left(W_\mu^\pm c^A + \frac{c_W}{s_W}W_\mu^\pm c^Z - A_\mu c^\pm - \frac{c_W}{s_W}Z_\mu c^\pm\right) \tag{110}$$

$$\delta_BZ_\mu = \partial_\mu c^Z + ie\frac{c_W}{s_W}\left(W_\mu^-c^+ - W_\mu^+c^-\right) \tag{111}$$

$$\delta_BA_\mu = \partial_\mu c^A + ie\left(W_\mu^-c^+ - W_\mu^+c^-\right) \tag{112}$$

In order to determines the BRS variation of scalars , we write Eq.98 by real components as

$$\delta_B \frac{1}{\sqrt{2}} \begin{pmatrix} i\chi_1 + \chi_2 \\ v + H - i\chi_3 \end{pmatrix} = \frac{i}{2} \begin{pmatrix} gc^3 + g'c^0 & g(c^1 - ic^2) \\ g(c^1 + ic^2) & -gc^3 + g'c^0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} i\chi_1 + \chi_2 \\ v + H - i\chi_3 \end{pmatrix} \tag{113}$$

and separate the real part and the imaginary part to get the following relations.

$$\delta_B \chi^{\pm} = \frac{e}{2s_W}[c^{\pm}(v+H) \mp ic^{\pm}\chi_3] \pm ie\left(c^A\chi^{\pm} + \frac{1-2s_W^2}{2s_W c_W}c^Z\chi^{\pm}\right) \tag{114}$$

$$\delta_B H = -\frac{e}{2s_W}(c^-\chi^+ + c^+\chi^-) - \frac{e}{2s_W c_W}c^Z\chi_3 \tag{115}$$

$$\delta_B \chi_3 = -i\frac{e}{2s_W}(c^-\chi^+ - c^+\chi^-) + \frac{e}{2s_W c_W}c^Z(v+H) \tag{116}$$

3.3 Gauge fixing by BRS symmetry

Now we are ready to determine the Lagrangian of gauge theory in quantum level.

- The tree Lagrangian in Sec.1, $L(gauge) + L(fermion) + L(scalar)$ is BRS invariant since the BRS transformation is obtained by the replacement in Eq.92.
- The explicit form of the BRS variation is determined before the introduction of gauge fixing terms. So the present method is independent of the way how gauge is fixed.
- We request that the total Lagrangian is BRS invariant. The gauge fixing is done by the addition of terms in Eq.89 to the tree Lagrangian. By the nilpotency of δ_B,

$$\delta_B(\bar{c}^a F^a) \tag{117}$$

  is BRS invariant. This becomes as follow:

$$\delta_B(\bar{c}^a F^a) = B^a F^a - \bar{c}^a \delta_B F^a. \tag{118}$$

  The first term gives one term in Eq.89 and the other term $B^a B^a$ is BRS invariant by Eq.101. (And still ξ is arbitrary.) Thus the Lagrangian

$$L = L(gauge) + L(fermion) + L(scalar) + \frac{\xi}{2}B^a B^a + \delta_B(\bar{c}^a F^a) \tag{119}$$

  is BRS invariant and includes gauge fixing terms.

- The total Lagrangian is

$$L = L(gauge) + L(fermion) + L(scalar) + L(G.F.) + L(F.P.) \tag{120}$$

  where

$$L(G.F.) = \frac{\xi}{2}B^a B^a + B^a F^a, \tag{121}$$

$$L(F.P.) = -\bar{c}^a \delta_B F^a. \tag{122}$$

  The latter name show that this term is first obtained by Faddeev-Popov through path integral quantization. Later, we discuss the renormalization. It should be noted that the gauge fixing terms are written in the renormalized fields.

3.4 Linear gauge fixing

Before the discussion on the non-linear gauge, we describe the conventional linear covariant gauge. Since the gauge fixing terms are to be written in renormalized physical fields, we use the suffix A, Z, W to specify terms.

The linear gauge fixing terms in Eq.88 form are as follows:

$$L(G.F.) = L^W(G.F.) + L^Z(G.F.) + L^A(G.F.) \tag{123}$$

$$\begin{aligned} L^W(G.F.) &= -\frac{1}{\xi_W}F^+F^-, \quad F^\pm = \partial^\mu W^\pm_\mu + \xi_W M_W \chi^\pm \\ L^Z(G.F.) &= -\frac{1}{2\xi_Z}(F^Z)^2, \quad F^Z = \partial^\mu Z_\mu + \xi_Z M_Z \chi_3 \\ L^A(G.F.) &= -\frac{1}{2\xi}(F^A)^2, \qquad F^A = \partial^\mu A_\mu \end{aligned} \tag{124}$$

We sum all bosonic bilinear terms.

1. Kinetic term for gauge fields, $L^{(2)}(gauge)$ in Eq.76
2. Kinetic term for scalar fields, $L^{(2)}(s, kin)$ in Eq.45
3. Bilinear term in the scalar potential, $L^{(2)}(pot)$ in Eq.70
4. $L(G.F.)$ (above)

Then we obtain the following results.

$$\begin{aligned} L^{(2)}(boson) = \; & -\frac{1}{2}\partial_{[\mu}W^-_{\nu]}\partial^{[\mu}W^{+\nu]} - \frac{1}{\xi_W}\partial^\mu W^-_\mu \partial^\nu W^+_\nu + M_W^2 W^-_\mu W^{+\mu} \\ & -\frac{1}{4}\partial_{[\mu}Z_{\nu]}\partial^{[\mu}Z^{\nu]} - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu)^2 + \frac{1}{2}M_Z^2(Z_\mu)^2 \\ & -\frac{1}{4}\partial_{[\mu}A_{\nu]}\partial^{[\mu}A^{\nu]} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 \\ & +\partial_\mu\chi^-\partial^\mu\chi^+ - \xi_W M_W^2\chi^-\chi^+ + \frac{1}{2}(\partial_\mu\chi_3)^2 - \frac{1}{2}\xi_Z M_Z^2\chi_3^2 + \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}M_H^2 H^2 \\ & +\frac{3T}{2v}H^2 + \frac{T}{2v}\chi_3^2 + \frac{T}{v}\chi^+\chi^- \end{aligned} \tag{125}$$

It will be useful to derive the propagators of gauge bosons here. We write

$$L^{(2)}(boson) = -W^-_\mu \mathcal{D}^{\mu\nu}_W W^+_\nu - \frac{1}{2}Z_\mu \mathcal{D}^{\mu\nu}_Z Z_\nu - \frac{1}{2}A_\mu \mathcal{D}^{\mu\nu}_A A_\nu + \cdots \tag{126}$$

where

$$\mathcal{D}^{\mu\nu}_W = -\partial_\alpha\partial^\alpha g^{\mu\nu} + \left(1 - \frac{1}{\xi_W}\right)\partial^\mu\partial^\nu - M_W^2 g^{\mu\nu}, \tag{127}$$

$$\mathcal{D}^{\mu\nu}_Z = -\partial_\alpha\partial^\alpha g^{\mu\nu} + \left(1 - \frac{1}{\xi_Z}\right)\partial^\mu\partial^\nu - M_Z^2 g^{\mu\nu}, \tag{128}$$

$$\mathcal{D}_A^{\mu\nu} = -\partial_\alpha\partial^\alpha g^{\mu\nu} + \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu. \tag{129}$$

The inverse of $\mathcal{D}$ is defined as

$$\mathcal{D}^{\mu\nu}(\mathcal{D}^{-1})_{\nu\rho} = g^\mu_\rho. \tag{130}$$

The inverse can be calculated solving $\mathcal{D}^{\mu\nu}(ag_{\nu\rho} + b\partial_\nu\partial_\rho) = g^\mu_\rho$. They are

$$(\mathcal{D}_W^{-1})_{\mu\nu} = -\frac{1}{\partial_\alpha\partial^\alpha + M_W^2}\left(g_{\mu\nu} - (1-\xi_W)\frac{\partial_\mu\partial_\nu}{\partial_\alpha\partial^\alpha + \xi_W M_W^2}\right), \tag{131}$$

$$(\mathcal{D}_Z^{-1})_{\mu\nu} = -\frac{1}{\partial_\alpha\partial^\alpha + M_Z^2}\left(g_{\mu\nu} - (1-\xi_Z)\frac{\partial_\mu\partial_\nu}{\partial_\alpha\partial^\alpha + \xi_W M_Z^2}\right), \tag{132}$$

$$(\mathcal{D}_A^{-1})_{\mu\nu} = -\frac{1}{\partial_\alpha\partial^\alpha}\left(g_{\mu\nu} - (1-\xi)\frac{\partial_\mu\partial_\nu}{\partial_\alpha\partial^\alpha}\right). \tag{133}$$

These functions $\mathcal{D}^{-1}$ are propagators of gauge bosons.

1. It should be noted that the gauge fixing terms are chosen so as to cancel the transition terms between W, Z and χ (e.g., $M_W\partial_\mu\chi^+W^{-\mu}$) which appear in $L^{(2)}(s, kin)$.
2. Higgs particle H is a real particle of mass M_H.
3. The poles of W, Z, and A are M_W, M_Z, and 0, respectively.
4. χ particles seem to appear, though their 'squared masses' are $\xi_W M_W^2$ and $\xi_Z M_Z^2$. The fact that the 'mass' is gauge dependent is the feature of unphysical particles.
5. If we take $\xi_W, \xi_Z \to \infty$, the 'mass' of χ's becomes infinity, so that χ's decouple from the real world. This is the unitary gauge. It corresponds to the case in which χ's are absorbed by redefinition of gauge fields by Eq.48. (Formally, there is no gauge fixing terms when $\xi_W, \xi_Z \to \infty$.)
6. If we take $\xi = \xi_W = \xi_Z = 1$, the numerator of propagators is proportional to $g_{\mu\nu}$. This helps the practical calculation and is called as 'tHooft-Feynman gauge.

3.5 Non-linear gauge fixing

Non-linear gauge is an extension of the last subsection and the common points are skipped here.

The gauge fixing terms in non-linear gauge is as follows:

$$\begin{aligned}
F^\pm &= \left(\partial^\mu \mp ie\tilde{\alpha}A^\mu \mp i\frac{ec_W}{s_W}\tilde{\beta}Z^\mu\right)W^\pm_\mu + \xi_W\left(M_W\chi^\pm + \frac{e}{2s_W}\tilde{\delta}H\chi^\pm \pm i\frac{e}{2s_W}\tilde{\kappa}\chi_3\chi^\pm\right) \\
F^Z &= \partial^\mu Z_\mu + \xi_Z\left(M_Z\chi_3 + \frac{e}{2s_W c_W}\tilde{\varepsilon}H\chi_3\right) \\
F^A &= \partial^\mu A_\mu
\end{aligned} \tag{134}$$

Here, $\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon}$ are non-linear gauge parameters specific to this gauge.

It can be seen that the linear terms are common to those in the linear gauge.

$$L(G.F.) = L^{(2)}(G.F.) + L^{(3)}(G.F.) + L^{(4)}(G.F.) \tag{135}$$

Since there is no change in the bilinear terms, $L^{(2)}(G.F.)$, the statements in the last subsection also hold here. However, this gauge includes new interaction terms which depend on $\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon}$. The change is confined only in the bosonic sector, and there is no change in the fermion interaction with bosons. Also there appears difference in the ghost sector as is discussed in the next subsection.

Non-linear gauge parameter dependent terms are as follows:

$$
\begin{aligned}
L^{(3)}(G.F.) = \; & ie\left(\frac{\tilde{\alpha}}{\xi_W}A^\mu + \frac{c_W}{s_W}\frac{\tilde{\beta}}{\xi_W}Z^\mu\right)(W^+_\mu \partial^\nu W^-_\nu - W^-_\mu \partial^\nu W^+_\nu) \\
& +ie\left(\tilde{\alpha}M_W A^\mu + \frac{c_W}{s_W}\tilde{\beta}M_W Z^\mu\right)(W^+_\mu \chi^- - W^-_\mu \chi^+) \\
& -\frac{e}{2s_W}\tilde{\delta}(\partial^\mu W^-_\mu \chi^+ + \partial^\mu W^+_\mu \chi^-)H - i\frac{e}{2s_W}\tilde{\varepsilon}(\partial^\mu W^-_\mu \chi^+ - \partial^\mu W^+_\mu \chi^-)\chi_3 \\
& -\frac{e}{2s_W c_W}\tilde{\kappa}\partial^\mu Z_\mu \chi_3 H \\
& -\xi_W \frac{e}{s_W}\tilde{\delta}M_W \chi^+\chi^- H - \xi_Z \frac{e}{2s_W c_W}\tilde{\varepsilon}M_Z \chi_3^2 H
\end{aligned} \tag{136}
$$

$$
\begin{aligned}
L^{(4)}(G.F.) = \; & -\frac{e^2}{\xi_W}[\tilde{\alpha}^2 A^\mu W^-_\mu A^\nu W^+_\nu + \tilde{\alpha}\tilde{\beta}(Z^\mu W^-_\mu A^\nu W^+_\nu + A^\mu W^-_\mu Z^\nu W^+_\nu) + \tilde{\beta}^2 Z^\mu W^-_\mu Z^\nu W^+_\nu] \\
& -ie^2\frac{1}{2s_W}\tilde{\alpha}\tilde{\delta}(A^\mu W^-_\mu \chi^+ - A^\mu W^+_\mu \chi^-)H - ie^2\frac{c_W}{2s_W^2}\tilde{\beta}\tilde{\delta}(Z^\mu W^-_\mu \chi^+ - Z^\mu W^+_\mu \chi^-)H \\
& +e^2\frac{1}{2s_W}\tilde{\alpha}\tilde{\kappa}(A^\mu W^-_\mu \chi^+ + A^\mu W^+_\mu \chi^-)\chi_3 + e^2\frac{c_W}{2s_W^2}\tilde{\beta}\tilde{\kappa}(Z^\mu W^-_\mu \chi^+ + Z^\mu W^+_\mu \chi^-)\chi_3 \\
& -\xi_W\frac{e^2}{4s_W^2}\tilde{\delta}^2 H^2\chi^+\chi^- - \xi_W\frac{e^2}{4s_W^2}\tilde{\kappa}^2\chi_3^2\chi^+\chi^- - \xi_Z\frac{e^2}{8s_W^2c_W^2}\tilde{\varepsilon}^2 H^2\chi_3^2
\end{aligned} \tag{137}
$$

As above, several gauge-dependent interaction terms are added. Sometimes this helps us to control the interaction. For an instance, we compare $L^{(3)}(G.F.)$ with $L^{(3,VVS)}(s,kin)$ in Eq.52. Then we find the followings:

$$\tilde{\alpha} = 1 \quad \Rightarrow \quad AW\chi \text{terms vanish} \tag{138}$$

$$\tilde{\beta} = -\frac{s_W^2}{c_W^2} \quad \Rightarrow \quad ZW\chi \text{terms vanish} \tag{139}$$

If we specify the non-linear gauge parameter to such value, we can reduce the number of Feynman diagrams for a process. (If we have a diagram including AWW vertex, we always have another diagram of the same structure but with $AW\chi$ vertex as long as the W line is an internal line.) This feature has been utilized to simplify the hand computation.

3.6 Ghost sector in the non-linear gauge

We have already known how to construct the ghost Lagrangian in Sec.3.3. The form is given by Eq.122. Corresponding to three gauge fixing conditions, we use ghost fields, $c^\pm, c^Z, c^A$, and anti-ghost fields as $\bar{c}^{\pm}, \bar{c}^{Z}, \bar{c}^{A}$, respectively. The calculation of $\delta_B F$ can be done using Eq.110 $\sim$ Eq.116.

$$L(F.P.) = L_W(F.P.) + L_Z(F.P.) + L_A(F.P.) \tag{140}$$

$$L_W(F.P.) = -\bar{c}^{+}\delta_B F^{-} - \bar{c}^{-}\delta_B F^{+},\ L_Z(F.P.) = -\bar{c}^{Z}\delta_B F^{Z},\ L_A(F.P.) = -\bar{c}^{A}\delta_B F^{A}. \tag{141}$$

$$L_A(F.P.) = L_A^{(2)}(F.P.) + L_A^{(3)}(F.P.) \tag{142}$$

$$L_A^{(2)}(F.P.) = -\bar{c}^{A}\partial_\mu\partial^\mu c^{A} \tag{143}$$

$$L_A^{(3)}(F.P.) = ie(\partial^\mu \bar{c}^{A})(W_\mu^{-}c^{+} - W_\mu^{+}c^{-}) \tag{144}$$

Here and in the following, we frequently use the replacement

$$\bar{c}\partial X \quad \rightarrow \quad -(\partial\bar{c})X$$

to put the derivative on an anti-ghost field as convention.

$$L_Z(F.P.) = L_Z^{(2)}(F.P.) + L_Z^{(3)}(F.P.) + L_Z^{(4)}(F.P.) \tag{145}$$

$$L_Z^{(2)}(F.P.) = -\bar{c}^{Z}\partial_\mu\partial^\mu c^{Z} - \xi_Z M_Z^2 \bar{c}^{Z}c^{Z} \tag{146}$$

$$\begin{aligned} L_Z^{(3)}(F.P.) = \ & ie\frac{c_W}{s_W}(\partial^\mu\bar{c}^{Z})(W_\mu^{-}c^{+} - W_\mu^{+}c^{-}) \\ & -\xi_Z\frac{e}{2s_W c_W}M_Z(1+\tilde{\varepsilon})\bar{c}^{Z}c^{Z}H \\ & +i\xi_Z\frac{e}{2s_W}M_Z(\bar{c}^{Z}c^{-}\chi^{+} - \bar{c}^{Z}c^{+}\chi^{-}) \end{aligned} \tag{147}$$

$$\begin{aligned} L_Z^{(4)}(F.P.) = \ & \xi_Z\frac{e^2}{4s_W^2c_W^2}\tilde{\varepsilon}\Big(-\bar{c}^{Z}c^{Z}H^2 + \bar{c}^{Z}c^{Z}\chi_3^2 \\ & +ic_W(\bar{c}^{Z}c^{-}\chi^{+} - \bar{c}^{Z}c^{+}\chi^{-})H + c_W(\bar{c}^{Z}c^{-}\chi^{+} + \bar{c}^{Z}c^{+}\chi^{-})\chi_3\Big) \end{aligned} \tag{148}$$

$$L_W(F.P.) = L_W^{+}(F.P.) + L_W^{-}(F.P.) = -\bar{c}^{+}\delta_B F^{-} - \bar{c}^{-}\delta_B F^{+} \tag{149}$$

Below only the first term (+ term) is shown. The second term is be given by the similar formulas.

$$L_W^{+}(F.P.) = L_W^{+(2)}(F.P.) + L_W^{+(3)}(F.P.) + L_W^{+(4)}(F.P.) \tag{150}$$

$$L_W^{+(2)}(F.P.) = -\bar{c}^{+}\partial_\mu\partial^\mu c^{-} - \xi_W M_W^2\bar{c}^{+}c^{-} \tag{151}$$

$$\begin{aligned} L_W^{+(3)}(F.P.) = \ & -ie\frac{c_W}{s_W}[(\partial^\mu\bar{c}^{+})W_\mu^{-}c^{Z} + \tilde{\beta}\bar{c}^{+}W_\mu^{-}(\partial^\mu c^{Z})] \ -ie[(\partial^\mu\bar{c}^{+})W_\mu^{-}c^{A} + \tilde{\alpha}\bar{c}^{+}W_\mu^{-}(\partial^\mu c^{A})] \\ & +ie\frac{c_W}{s_W}[(\partial^\mu\bar{c}^{+})Z_\mu c^{-} - \tilde{\beta}\bar{c}^{+}Z_\mu(\partial^\mu c^{-})] \quad +ie[(\partial^\mu\bar{c}^{+})A_\mu c^{-} - \tilde{\alpha}\bar{c}^{+}A_\mu(\partial^\mu c^{-})] \\ & -\xi_W\frac{e}{2s_W}M_W(1+\tilde{\delta})\bar{c}^{+}c^{-}H \quad -i\xi_W\frac{e}{2s_W}M_W(1-\tilde{\kappa})\bar{c}^{+}c^{-}\chi_3 \\ & +i\xi_W\frac{e}{2s_W c_W}\left(1 - 2s_W^2 + \tilde{\kappa}\right)M_W\bar{c}^{+}c^{Z}\chi^{-} \quad +i\xi_W e M_W\bar{c}^{+}c^{A}\chi^{-} \end{aligned} \tag{152}$$

$$
\begin{aligned}
L_W^{+(4)}(F.P.) = \;& e^2\tilde{\alpha}\left(-\frac{c_W}{s_W}\bar{c}^+c^Z A_\mu W^{-\mu} - \bar{c}^+c^A A_\mu W^{-\mu} + \bar{c}^+c^- A_\mu A^\mu + \frac{c_W}{s_W}\bar{c}^+c^- A_\mu Z^\mu\right) \\
& e^2\frac{c_W}{s_W}\tilde{\beta}\left(-\frac{c_W}{s_W}\bar{c}^+c^Z Z_\mu W^{-\mu} - \bar{c}^+c^A Z_\mu W^{-\mu} + \frac{c_W}{s_W}\bar{c}^+c^- Z_\mu Z^\mu + \bar{c}^+c^- Z_\mu A^\mu\right) \\
& +e^2\left(\tilde{\alpha} + \frac{c_W^2}{s_W^2}\tilde{\beta}\right)(-\bar{c}^+c^- W_\mu^+ W^{-\mu} + \bar{c}^+c^+ W_\mu^- W^{-\mu}) \\
& +\xi_W\frac{e^2}{2s_W}\tilde{\delta}\left(-\frac{1}{2s_W}\bar{c}^+c^- H^2 - i\frac{1}{2s_W}\bar{c}^+c^- H\chi_3 + i\frac{1-2s_W^2}{2s_W c_W}\bar{c}^+c^Z H\chi^- + i\bar{c}^+c^A H\chi^-\right) \\
& +\xi_W\frac{e^2}{2s_W}\tilde{\kappa}\left(+i\frac{1}{2s_W}\bar{c}^+c^- H\chi_3 - \frac{1}{2s_W}\bar{c}^+c^-\chi_3^2 + \frac{1-2s_W^2}{2s_W c_W}\bar{c}^+c^Z\chi_3\chi^- + \bar{c}^+c^A\chi_3\chi^-\right) \\
& +\xi_W\frac{e^2}{4s_W^2}\tilde{\delta}\left(+\bar{c}^+c^-\chi^+\chi^- + \bar{c}^+c^+(\chi^-)^2 + \frac{1}{c_W}\bar{c}^+c^Z\chi_3\chi^-\right) \\
& +\xi_W\frac{e^2}{4s_W^2}\tilde{\kappa}\left(+\bar{c}^+c^-\chi^+\chi^- - \bar{c}^+c^+(\chi^-)^2 + i\frac{1}{c_W}\bar{c}^+c^Z H\chi^-\right)
\end{aligned}
\tag{153}
$$

4 Lagrangian and parameters

4.1 Full Lagrangian

First, we summarize the total tree Lagrangian of the standard model in the non-linear gauge.

1.free part

gauge boson	$L^{(2)}(boson)$	Eq.125 (see also Eq.126)
fermion	$L(f,kin) + L^{(2)}(s-f)$	Eq.80, Eq.58
higgs and χ	$L^{(2)}(boson)$	Eq.125
ghost	$L^{(2)}_{A,Z,W}(F.P.)$	Eq.143, Eq.146, Eq.151

2.Interaction part, bosonic

$L^{(3)}(gauge)$	Eq.77	vvv
$L^{(4)}(gauge)$	Eq.78	vvvv
$L^{(3,VVS)}(s,kin)$	Eq.52	vvs
$L^{(3,SSV)}(s,kin)$	Eq.51	ssv
$L^{(4)}(s,kin)$	Eq.55	vvss
$L^{(3)}(pot)$	Eq.71	sss
$L^{(4)}(pot)$	Eq.72	ssss
$L^{(3)}(G.F.)$	Eq.136	vvv, vvs, ssv
$L^{(4)}(G.F.)$	Eq.137	vvvv, vvss, ssss

3.Interaction part, fermionic

$L(f, g-f)$	Eq.81	ffv
$L^{(3)}(s-f)$	Eq.60	ffs

4.Interaction part, ghost

$L^{(3)}_{A,Z,W}(F.P.)$	Eq.144, Eq.147, Eq.152	ggv, ggs
$L^{(4)}_{Z,W}(F.P.)$	Eq.148, Eq.153	ggvv, ggss

5.Tadpole term

TH term	Eq.63	TH

4.2 Parameters

The parameters in the theory are as follows:

$$g, g', v, \mu^2, \lambda, \{f_U, f_D\} \tag{154}$$

These are replaced by the following physical parameters.

$$e, M_W, M_Z, M_H^2, T, \{m_f\} \tag{155}$$

The first set of parameters are written by the physical parameters. (Eq.67, Eq.68, Eq.38, Eq.42, Eq.46, Eq.59)

We sometimes use the notations s_W, c_W, v just as the shorthand of

$$s_W = \frac{\sqrt{M_Z^2 - M_W^2}}{M_Z},\ c_W = \frac{M_W}{M_Z},\ v = \frac{2 s_W M_W}{e} = \frac{2\sqrt{M_Z^2 - M_W^2}\, M_W}{e M_Z}. \tag{156}$$

Numerical values of these parameters given in 1999 PDG report are as follows:

$e(= e/\sqrt{\varepsilon_0 c\hbar})$	0.3029
(α^{-1})	137.036
M_W	80.41 ± 0.10
M_Z	91.187 ± 0.007
$(c_W = M_W/M_Z)$	0.8818
$(s_W = \sqrt{M_Z^2 - M_W^2}/M_Z)$	0.4716
$(v = 2 s_W M_W/e)$	250.5

In the tree level, as in Eq.67,

$$T = v(\mu^2 - \lambda v^2). \tag{157}$$

The parameter T is not an exact independent parameter. Discussed in Sec.2.1 and in Sec.2.3. If we specify that the v is potential minimum, then $T = 0$ is required. This means that under the minimum condition, either of μ^2 or λ is not independent.

5 Perturbation

In this section, we briefly review the perturbative method in the quantum field theory. For simplicity, we deal with a scalar field $\phi(x)$ of mass m.

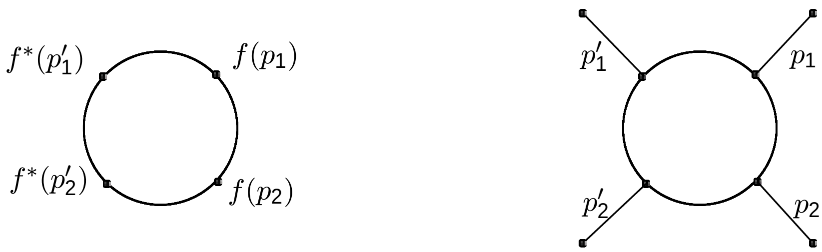
The S-matrix element describes the interaction of fundamental particles. By the LSZ reduction formula, the S-matrix for the process $(p_1, p_2, \cdots, p_n) \to (p_1', p_2', \cdots, p_m')$ is

$$\begin{aligned} S_{fi} &=< f|i>=< p_1', p_2', \cdots, p_m'|p_1, p_2, \cdots, p_n > \\ &= \prod_{k=1}^{m} \left(i \int d^4 y_k f^*(p_k') \mathcal{D}_{y_k} \right) \prod_{j=1}^{n} \left(i \int d^4 x_j f(p_j) \mathcal{D}_{x_k} \right) G^{n+m}(y_1, y_2, \cdots, y_m, x_1, x_2, \cdots, x_n) \end{aligned} \tag{158}$$

where G^N is the N-point Green function

$$G^N(x_1, x_2, \cdots, x_N) =< 0|T[\boldsymbol{\phi}(x_1)\boldsymbol{\phi}(x_2)\cdots\boldsymbol{\phi}(x_N)]| >, \tag{159}$$

$\mathcal{D}_x = \partial_\mu \partial^\mu + m^2$ is the inverse propagator, and $f(k)$ is the wave function for the external particle of momentum k. In the definition of Green function, $\boldsymbol{\phi}$ is the renormalized Heisenberg field ($\sqrt{Z}\boldsymbol{\phi} = \phi_H$ where ϕ_H is the Heisenberg field) and T stands for the T-product. The relation of S-matrix and Green function is depicted in Fig.1.



(a) S_{fi} (b) G^n

Figure 1: S-matrix and Green function for a two-to-two process

Next, we introduce the generating functional of Green functions.

$$Z[j] =< 0|T \exp[i \int d^4x J(x)\boldsymbol{\phi}(x)]|0 > \tag{160}$$

Here the source, $J(x)$, is c-number quantity. If we differentiate $Z[J]$ by $J(x)$, we obtain the Green functions. This is the reason we call $Z[J]$ as generating functional of G^n.

$$G^n(x_1, x_2, \cdots, x_n) = \left(-i\frac{\delta}{\delta J(x_1)}\right)\left(-i\frac{\delta}{\delta J(x_2)}\right)\cdots\left(-i\frac{\delta}{\delta J(x_n)}\right) Z[J]\Big|_{J=0} \tag{161}$$

Sources are set to be 0 after taking derivative.

When we use the path-integral method, $Z[J]$ is written as follows.

$$Z[J] = N \int [d\phi] \exp\left[i \int d^4x \{L[\phi(x)] + J(x)\phi(x)\} \right] \tag{162}$$

where N is a (infinite) constant, $\phi(x)$ is a classical (c-number) field corresponds to $\boldsymbol{\phi}(x)$, and $[d\phi] = \prod_x d\phi(x)$ is the (hyper-)product of integration by the field. The Lagrangian is given by

$$L[\phi] = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\phi^2 + L^I[\phi] = -\frac{1}{2}\phi\mathcal{D}\phi + L^I[\phi] \tag{163}$$

where L^I is the interaction Lagrangian.

The evaluation of Eq.162 is done by the following method. We replace ϕ in L^I by the derivative δ/dJ.

$$Z[J] = N \exp\left[i \int d^4x L^I[-i\delta/dJ]\right] \int [d\phi] \exp\left[i \int d^4x\{-\frac{1}{2}\phi\mathcal{D}\phi + J(x)\phi(x)\}\right] \tag{164}$$

The integral $[d\phi]$ can be done by the Gaussian integral.

$$\begin{aligned} &\int [d\phi] \exp\left[i \int d^4x\{-\frac{1}{2}\phi\mathcal{D}\phi + J(x)\phi(x)\}\right] \\ &= \int [d\phi] \exp\left[i \int d^4x\{-\frac{1}{2}(\phi - J\mathcal{D}^{-1})\mathcal{D}(\phi - \mathcal{D}^{-1}J) + \frac{1}{2}J\mathcal{D}^{-1}J\}\right] \\ &= (const.) \exp\left[i \int d^4x \frac{1}{2} J\mathcal{D}^{-1}J\right] \end{aligned} \tag{165}$$

Thus,

$$Z[J] = N' \exp\left[i \int d^4x L^I[-i\delta/dJ]\right] \exp\left[i \int d^4x \frac{1}{2} J\mathcal{D}^{-1}J\right]. \tag{166}$$

Here the Gaussian integral is performed by understanding

$$\int d^4x \frac{1}{2}\phi\mathcal{D}\phi = \int d^4x d^4y \frac{1}{2}\phi(x)\mathcal{D}(x,y)\phi(y), \quad (\mathcal{D}(x,y) = \mathcal{D}(x)\delta(x-y))$$

and

$$\int d^4x \frac{1}{2} J\mathcal{D}^{-1}J = \int d^4x d^4y \frac{1}{2} J(x)\mathcal{D}^{-1}(x,y)J(y), \quad \left(\int d^4z \mathcal{D}(x,z)\mathcal{D}^{-1}(z,y) = \delta(x-y)\right)$$

where $\mathcal{D}^{-1}(x,y) = \Delta_F(x,y)$ is the propagator.

The evaluation of Eq.166 is straightforward and it gives conventional Feynman rules. For an explicit demonstration, we assume

$$L^I[\phi] = g\phi^3. \tag{167}$$

Then we calculate the 2-point Green function of $O(g^2)$.

$$G^2(x,y) = \left(-i\frac{\delta}{\delta J_x}\right)\left(-i\frac{\delta}{\delta J_y}\right) N' \frac{1}{2}(ig)^2 \left(-i\frac{\delta}{\delta J_a}\right)^3 \left(-i\frac{\delta}{\delta J_b}\right)^3 \exp\left[i \int d^4x \frac{1}{2} J\mathcal{D}^{-1}J\right]. \tag{168}$$

Here we expand the interaction part and keep g^2 terms only. After the derivative, we put $J = 0$, so that the non-zero contribution comes 4-th order term of the $\exp[\cdots]$.

$$\begin{aligned} G^2(x,y) = \; &\left(-i\frac{\delta}{\delta J_x}\right)\left(-i\frac{\delta}{\delta J_y}\right) N' \frac{1}{2}(ig)^2 \left(-i\frac{\delta}{\delta J_a}\right)^3 \left(-i\frac{\delta}{\delta J_b}\right)^3 \\ &\times \frac{i^4}{4!}\frac{1}{2} J_1\Delta_F^{12}J_2 \frac{1}{2} J_3\Delta_F^{34}J_4 \frac{1}{2} J_5\Delta_F^{56}J_6 \frac{1}{2} J_6\Delta_F^{78}J_8 \end{aligned} \tag{169}$$

In the above, $J_k = J(x_k)$, $\Delta_F^{jk} = \Delta(x_j, x_k)$, and the integral for $a, b, 1, \sim 8$ is not shown explicitly. Expanding this equation, we obtain various terms which are shown in Fig.2.

The comparison between Fig.2 and terms Eq.169 tells the correspondence:

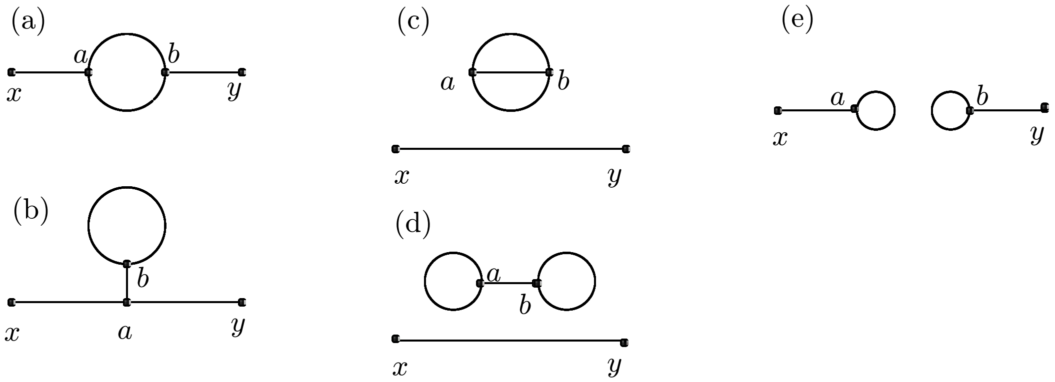


Figure 2: Diagrams for G^2 in $O(g^2)$

internal vertex	$\cdots$	ig
connection line	$\cdots$	Δ_F

In this manner, the Feynman rules can be constructed. Here, we skip the discussion on the statistical weight for identical particles.

As is shown in Fig.2, G includes several unnecessary terms. Terms (a) and (b) are to be separated from others. We introduce the generating functional for connected part.

$$iW[J] = \log Z[J] \tag{170}$$

In case on G^2 in $O(g^2)$,

$$\frac{\delta^2 W[J]}{\delta J(x)\delta J(y)} = \frac{1}{Z[J]}\frac{\delta^2 Z[J]}{\delta J(x)\delta J(y)} - \frac{1}{Z^2[J]}\frac{\delta Z[J]}{\delta J(x)}\frac{\delta Z[J]}{\delta J(y)} \tag{171}$$

Here, $1/Z[J]$ in the first term cancels (c) and (d) in Fig.2 and the second term cancels (e). The constant N vanishes when we use $W[J]$ instead of $Z[J]$.

6 Feynman rules

Basic Feynman rules follow the so-called Kyoto convention. A particle at the endpoint enters into the vertex. For an instance, if a line is denoted as W^+, then the line shows either the incoming W^+ or the outgoing W^-. The momentum assigned to a particle is defined as inward except for the case of a ghost particle for which the momentum is defined along the flow of ghost number.

6.1 Construction of Feynman rules

The relation between the rules in the following subsections and those in the conventional textbooks are as follows.

	Kyoto	Kyoto $\to$ textbook	textbook
Amplitude	T	$\times i$	iT
Propagator	$i < \phi\phi >$	$\times - i$	$< \phi\phi >$
Vertex	L_{int}	$\times i$	iL_{int}
Loop	$\int \frac{d^4 l}{i(2\pi)^4}$	$\times i$	$\int \frac{d^4 l}{(2\pi)^4}$
External line		(same)	

When we write down the vertex rules from L_{int}, two points are to be kept in mind.

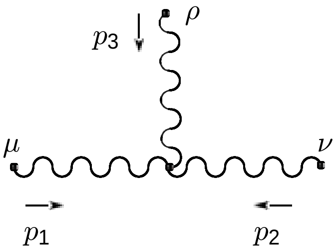
$$\text{Derivative} \qquad \partial_\mu \phi \quad \Rightarrow \quad -ip_\mu \tag{172}$$

$$\text{Identical fields} \qquad \phi^n \quad \Rightarrow n! \tag{173}$$

6.2 Propagators

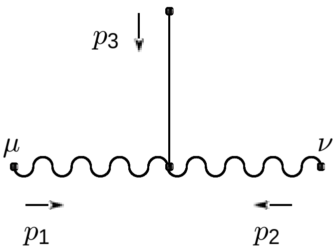
$W^\pm$	$\frac{1}{k^2 - M_W^2}\left(g_{\mu\nu} - (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right)$	Eq.131
Z	$\frac{1}{k^2 - M_Z^2}\left(g_{\mu\nu} - (1-\xi_Z)\frac{k_\mu k_\nu}{k^2 - \xi_W M_Z^2}\right)$	Eq.132
A	$\frac{1}{k^2}\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)$	Eq.133
f	$\frac{-1}{\gamma_\mu k^\mu - m_f}$	Eq.80, Eq.58
H	$\frac{-1}{k^2 - M_H^2}$	Eq.125
$\chi^\pm$	$\frac{-1}{k^2 - \xi_W M_W^2}$	Eq.125
χ_3	$\frac{-1}{k^2 - \xi_Z M_Z^2}$	Eq.125
$c^\pm$	$\frac{-1}{k^2 - \xi_W M_W^2}$	Eq.151
c^Z	$\frac{-1}{k^2 - \xi_Z M_Z^2}$	Eq.146
c^A	$\frac{-1}{k^2}$	Eq.143

6.3 Vector-Vector-Vector



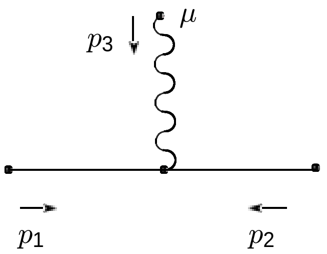
p_1 (μ)	p_2 (ν)	p_3 (ρ)	
W^-	W^+	A	$e\Big[g^{\mu\nu}(p_1-p_2)^\rho$ $+(1+\tilde{\alpha}/\xi_W)(p_3^\nu g^{\mu\rho}-p_3^\mu g^{\nu\rho})$ $+(1-\tilde{\alpha}/\xi_W)(p_2^\mu g^{\nu\rho}-p_1^\nu g^{\mu\rho})\Big]$
W^-	W^+	Z	$e\dfrac{c_W}{s_W}\Big[g^{\mu\nu}(p_1-p_2)^\rho$ $+(1+\tilde{\beta}/\xi_W)(p_3^\nu g^{\mu\rho}-p_3^\mu g^{\nu\rho})$ $+(1-\tilde{\beta}/\xi_W)(p_2^\mu g^{\nu\rho}-p_1^\nu g^{\mu\rho})\Big]$

6.4 Vector-Vector-Scalar



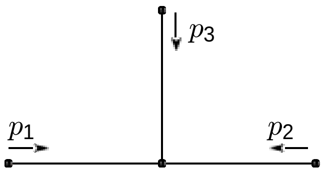
p_1 (μ)	p_2 (ν)	p_3	
$W^\pm$	A	$\chi^\mp$	$\mp ieM_W(1-\tilde{\alpha})g^{\mu\nu}$
$W^\pm$	Z	$\chi^\mp$	$\pm ie\dfrac{1}{s_W c_W}M_W\left(1-c_W^2(1-\tilde{\beta})\right)g^{\mu\nu}$
W^-	W^+	H	$e\dfrac{1}{s_W}M_W g^{\mu\nu}$
Z	Z	H	$e\dfrac{1}{s_W c_W^2}M_W g^{\mu\nu}$

6.5 Scalar-Scalar-Vector



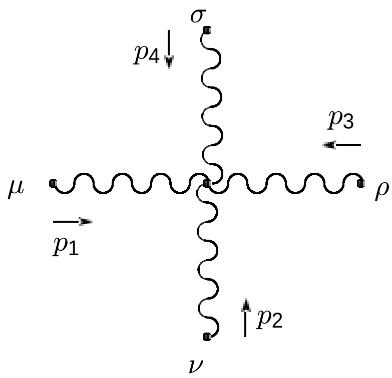
p_1	p_2	p_3 (μ)	
H	$\chi^{\mp}$	$W^{\pm}$	$ie\frac{1}{2s_W}\left[(1-\tilde{\delta})p_2^{\mu}-(1+\tilde{\delta})p_1^{\mu}\right]$
χ_3	$\chi^{\mp}$	$W^{\pm}$	$\pm e\frac{1}{2s_W}\left[(1-\tilde{\kappa})p_2^{\mu}-(1+\tilde{\kappa})p_1^{\mu}\right]$
χ^-	χ^+	A	$e(p_2-p_1)^{\mu}$
χ^-	χ^+	Z	$e\frac{c_W^2-s_W^2}{2s_Wc_W}(p_2-p_1)^{\mu}$
H	χ_3	Z	$ie\frac{1}{2s_Wc_W}\left[(1-\tilde{\varepsilon})p_2^{\mu}-(1+\tilde{\varepsilon})p_1^{\mu}\right]$

6.6 Scalar-Scalar-Scalar



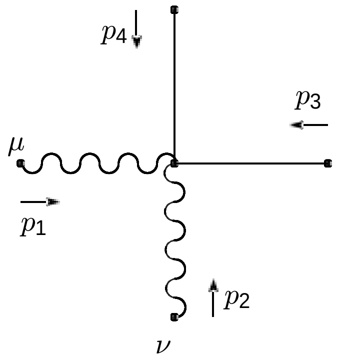
p_1	p_2	p_3	
H	H	H	$-e\frac{3}{2s_WM_W}M_H^2$
H	χ^-	χ^+	$-e\frac{1}{2s_WM_W}(M_H^2+2\tilde{\delta}M_W^2\cdot\xi_W)$
H	χ_3	χ_3	$-e\frac{1}{2s_WM_W}(M_H^2+2\tilde{\varepsilon}M_Z^2\cdot\xi_Z)$

6.7 Vector-Vector-Vector-Vector



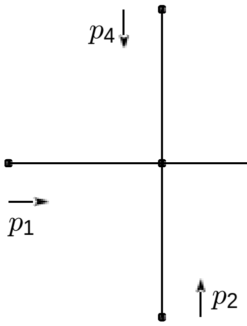
$p_1\ (\mu)$	$p_2\ (\nu)$	$p_3\ (\rho)$	$p_4\ (\sigma)$
W^+	W^-	A	A
$e^2\left[-2g^{\mu\nu}g^{\rho\sigma}+(1-\tilde{\alpha}^2/\xi_W)(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho})\right]$			
W^+	W^-	A	Z
$e^2\frac{c_W}{s_W}\left[-2g^{\mu\nu}g^{\rho\sigma}+(1-\tilde{\alpha}\tilde{\beta}/\xi_W)(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho})\right]$			
W^+	W^-	Z	Z
$e^2\frac{c_W^2}{s_W^2}\left[-2g^{\mu\nu}g^{\rho\sigma}+(1-\tilde{\beta}^2/\xi_W)(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho})\right]$			
W^+	W^-	W^-	W^+
$-e^2\frac{1}{s_W^2}\left[-2g^{\mu\sigma}g^{\nu\rho}+(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\nu}g^{\rho\sigma})\right]$			

6.8 Vector-Vector-Scalar-Scalar



p_1 (μ)	p_2 (ν)	p_3	p_4	
A	$W^\pm$	H	$\chi^\mp$	$\mp ie^2 \frac{1}{2s_W}(1-\tilde{\alpha}\tilde{\delta})g^{\mu\nu}$
A	$W^\pm$	χ_3	$\chi^\mp$	$-e^2 \frac{1}{2s_W}(1-\tilde{\alpha}\tilde{\kappa})g^{\mu\nu}$
Z	$W^\pm$	H	$\chi^\mp$	$\pm ie^2 \frac{1}{2s_W^2 c_W}\left(1-c_W^2(1-\tilde{\beta}\tilde{\delta})\right) g^{\mu\nu}$
Z	$W^\pm$	χ_3	$\chi^\mp$	$e^2 \frac{1}{2s_W^2 c_W}\left(1-c_W^2(1-\tilde{\beta}\tilde{\kappa})\right) g^{\mu\nu}$
A	A	χ^+	χ^-	$2e^2 g^{\mu\nu}$
Z	A	χ^+	χ^-	$2e^2 \frac{c_W^2 - s_W^2}{2s_W c_W} g^{\mu\nu}$
Z	Z	χ^+	χ^-	$2e^2 \left(\frac{c_W^2 - s_W^2}{2s_W c_W}\right)^2 g^{\mu\nu}$
W^+	W^-	H	H	$e^2 \frac{1}{2s_W^2} g^{\mu\nu}$
W^+	W^-	χ_3	χ_3	$e^2 \frac{1}{2s_W^2} g^{\mu\nu}$
W^+	W^-	χ^-	χ^+	$e^2 \frac{1}{2s_W^2} g^{\mu\nu}$
Z	Z	H	H	$e^2 \frac{1}{2s_W^2 c_W^2} g^{\mu\nu}$
Z	Z	χ_3	χ_3	$e^2 \frac{1}{2s_W^2 c_W^2} g^{\mu\nu}$

6.9 Scalar-Scalar-Scalar-Scalar

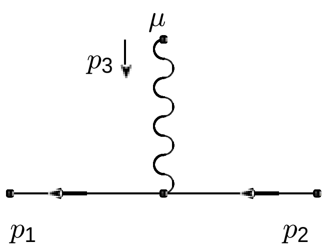


p_1	p_2	p_3	p_4	
H	H	H	H	$-e^2\dfrac{3M_H^2}{4s_W^2 M_W^2}$
χ_3	χ_3	χ_3	χ_3	$-e^2\dfrac{3M_H^2}{4s_W^2 M_W^2}$
$\chi^\pm$	$\chi^\mp$	$\chi^\mp$	$\chi^\pm$	$-e^2\dfrac{M_H^2}{2s_W^2 M_W^2}$
H	H	χ_3	χ_3	$-e^2\dfrac{M_H^2+2\tilde{\varepsilon}^2 M_Z^2\cdot\xi_Z}{4s_W^2 M_W^2}$
H	H	χ^+	χ^-	$-e^2\dfrac{M_H^2+2\tilde{\delta}^2 M_W^2\cdot\xi_W}{4s_W^2 M_W^2}$
χ^+	χ^-	χ_3	χ_3	$-e^2\dfrac{M_H^2+2\tilde{\kappa}^2 M_W^2\cdot\xi_W}{4s_W^2 M_W^2}$

6.10 Fermion-Fermion-Vector

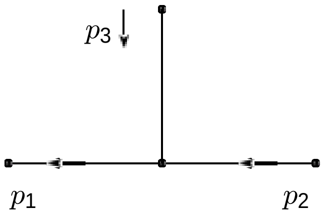
Mixing of fermion is not explicitly shown here. Though it is not explicitly written, one should mind that a quark has color degree of freedom.

f		I_3	Q_f		I_3	Q_f
U	u, c, t	$\frac{1}{2}$	$\frac{2}{3}$	ν_e, ν_μ, ν_τ	$\frac{1}{2}$	0
D	d, s, b	$-\frac{1}{2}$	$-\frac{1}{3}$	e, μ, τ	$-\frac{1}{2}$	-1



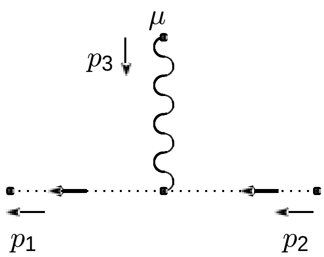
p_1	p_2	p_3 (μ)	
$\bar{f}$	f	A	$eQ_f\gamma^\mu$
$\bar{f}$	f	Z	$e\dfrac{1}{2s_W c_W}\gamma^\mu\left(I_3(1-\gamma_5)-2s_W^2 Q_f\right)$
$\bar{U}/\bar{D}$	D/U	W^+/W^-	$e\dfrac{1}{2\sqrt{2}s_W}\gamma^\mu(1-\gamma_5)$

6.11 Fermion-Fermion-Scalar



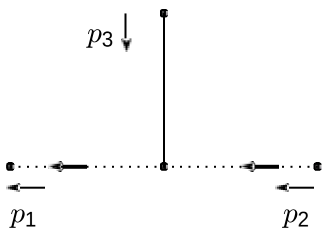
p_1	p_2	p_3	
$\bar{f}$	f	H	$-e\frac{1}{2s_W}\frac{m_f}{M_W}$
$\bar{U}/\bar{D}$	U/D	χ_3	$(-/+)ie\frac{1}{2s_W}\frac{m_f}{M_W}\,\gamma_5$
$\bar{U}$	D	χ^+	$-ie\frac{1}{2\sqrt{2}s_W}\frac{1}{M_W}\left[(m_D-m_U)+(m_D+m_U)\gamma_5\right]$
$\bar{D}$	U	χ^-	$-ie\frac{1}{2\sqrt{2}s_W}\frac{1}{M_W}\left[(m_U-m_D)+(m_U+m_D)\gamma_5\right]$

6.12 Ghost-Ghost-Vector



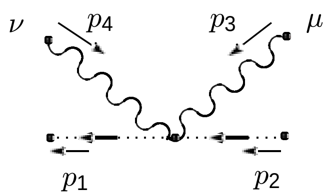
p_1	p_2	p_3 (μ)	
$\bar{c}^A$	$c^\mp$	$W^\pm$	$\pm e p_1^\mu$
$\bar{c}^Z$	$c^\mp$	$W^\pm$	$\pm e\frac{c_W}{s_W}p_1^\mu$
$\bar{c}^\mp$	c^A	$W^\pm$	$\mp e(p_1^\mu-\tilde{\alpha}p_2^\mu)$
$\bar{c}^\mp$	c^Z	$W^\pm$	$\mp e\frac{c_W}{s_W}(p_1^\mu-\tilde{\beta}p_2^\mu)$
$\bar{c}^\mp$	$c^\pm$	A	$\pm e(p_1^\mu+\tilde{\alpha}p_2^\mu)$
$\bar{c}^\mp$	$c^\pm$	Z	$\pm e\frac{c_W}{s_W}(p_1^\mu+\tilde{\beta}p_2^\mu)$

6.13 Ghost-Ghost-Scalar



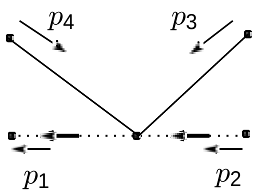
p_1	p_2	p_3	
$\bar{c}^Z$	c^Z	H	$-e\frac{1}{2s_W c_W^2}(1+\tilde{\varepsilon})M_W \cdot \xi_Z$
$\bar{c}^Z$	$c^\mp$	$\chi^\pm$	$\pm ie\frac{1}{2s_W c_W}M_W \cdot \xi_Z$
$\bar{c}^\mp$	c^A	$\chi^\pm$	$\mp ieM_W \cdot \xi_W$
$\bar{c}^\mp$	c^Z	$\chi^\pm$	$\mp ie\frac{1}{2s_W c_W}(c_W^2 - s_W^2 + \tilde{\kappa})M_W \cdot \xi_W$
$\bar{c}^\mp$	$c^\pm$	H	$-e\frac{1}{2s_W}(1+\tilde{\delta})M_W \cdot \xi_W$
$\bar{c}^\mp$	$c^\pm$	χ_3	$\pm ie\frac{1}{2s_W}(1-\tilde{\kappa})M_W \cdot \xi_W$

6.14 Ghost-Ghost-Vector-Vector



p_1	p_2	p_3 (μ)	p_4 (ν)	
$\bar{c}^\mp$	c^A	A	$W^\pm$	$-e^2\tilde{\alpha}g^{\mu\nu}$
$\bar{c}^\mp$	c^A	Z	$W^\pm$	$-e^2\frac{c_W}{s_W}\tilde{\beta}g^{\mu\nu}$
$\bar{c}^\mp$	c^Z	A	$W^\pm$	$-e^2\frac{c_W}{s_W}\tilde{\alpha}g^{\mu\nu}$
$\bar{c}^\mp$	c^Z	Z	$W^\pm$	$-e^2\frac{c_W^2}{s_W^2}\tilde{\beta}g^{\mu\nu}$
$\bar{c}^\mp$	$c^\pm$	$W^\mp$	$W^\pm$	$-e^2\left(\tilde{\alpha}+\frac{c_W^2}{s_W^2}\tilde{\beta}\right)g^{\mu\nu}$
$\bar{c}^\mp$	$c^\mp$	$W^\pm$	$W^\pm$	$2e^2\left(\tilde{\alpha}+\frac{c_W^2}{s_W^2}\tilde{\beta}\right)g^{\mu\nu}$
$\bar{c}^\mp$	$c^\pm$	A	A	$2e^2\tilde{\alpha}g^{\mu\nu}$
$\bar{c}^\mp$	$c^\pm$	Z	A	$e^2\frac{c_W}{s_W}(\tilde{\alpha}+\tilde{\beta})g^{\mu\nu}$
$\bar{c}^\mp$	$c^\pm$	Z	Z	$2e^2\frac{c_W^2}{s_W^2}\tilde{\beta}g^{\mu\nu}$

6.15 Ghost-Ghost-Scalar-Scalar



p_1	p_2	p_3 (μ)	p_4 (ν)	
$\bar{c}^Z$	c^Z	H	H	$-e^2 \frac{1}{2s_W^2 c_W^2} \tilde{\varepsilon} \cdot \xi_Z$
$\bar{c}^Z$	c^Z	χ_3	χ_3	$e^2 \frac{1}{2s_W^2 c_W^2} \tilde{\varepsilon} \cdot \xi_Z$
$\bar{c}^Z$	$c^\pm$	$\chi^\mp$	H	$\mp ie^2 \frac{1}{4s_W^2 c_W} \tilde{\varepsilon} \cdot \xi_Z$
$\bar{c}^Z$	$c^\pm$	$\chi^\mp$	χ_3	$e^2 \frac{1}{4s_W^2 c_W} \tilde{\varepsilon} \cdot \xi_Z$
$\bar{c}^\mp$	c^A	$\chi^\pm$	H	$\mp ie^2 \frac{1}{2s_W} \tilde{\delta} \cdot \xi_W$
$\bar{c}^\mp$	c^A	$\chi^\pm$	χ_3	$e^2 \frac{1}{2s_W} \tilde{\kappa} \cdot \xi_W$
$\bar{c}^\mp$	c^Z	$\chi^\pm$	H	$\mp ie^2 \frac{1}{4s_W^2 c_W} \left(\tilde{\kappa} + \tilde{\delta}(c_W^2 - s_W^2)\right) \cdot \xi_W$
$\bar{c}^\mp$	c^Z	$\chi^\pm$	χ_3	$e^2 \frac{1}{4s_W^2 c_W} \left(\tilde{\delta} + \tilde{\kappa}(c_W^2 - s_W^2)\right) \cdot \xi_W$
$\bar{c}^\mp$	$c^\pm$	H	H	$-e^2 \frac{1}{2s_W^2} \tilde{\delta} \cdot \xi_W$
$\bar{c}^\mp$	$c^\pm$	χ_3	χ_3	$-e^2 \frac{1}{2s_W^2} \tilde{\kappa} \cdot \xi_W$
$\bar{c}^\mp$	$c^\pm$	χ_3	H	$\mp ie^2 \frac{1}{4s_W^2} (\tilde{\kappa} - \tilde{\delta}) \cdot \xi_W$
$\bar{c}^\mp$	$c^\pm$	χ^-	χ^+	$e^2 \frac{1}{4s_W^2} (\tilde{\delta} + \tilde{\kappa}) \cdot \xi_W$
$\bar{c}^\mp$	$c^\mp$	$\chi^\pm$	$\chi^\pm$	$-e^2 \frac{1}{2s_W^2} (\tilde{\kappa} - \tilde{\delta}) \cdot \xi_W$

7 Renormalization

7.1 Renormalization constants

In this section, we discuss the renormalization of the theory. We implicitly assume the one-loop renormalization.

1. All fields and couplings appeared in the last sections are bare quantities. In order to denote explicitly, we put underline to the bare quantity. Renormalized quantities are shown without underline.

2. The field renormalization constant is defined as

$$\underline{\phi} = \sqrt{Z_\phi}\phi = (1 + \delta Z_\phi^{1/2})\phi. \tag{174}$$

Since we use the perturbative expansion, and that we perform the explicit study in the one-loop order, $\delta Z_\phi^{1/2}$ is a small quantity of $O(e^2)$ though it generally includes a divergent factor.

For the mixing fields, $Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + O(e^2)$, so that the off diagonal components has no "1".

There can be another convention for δZ such that

$$\underline{\phi} = \sqrt{Z_\phi}\phi = \left(1 + \frac{1}{2}\delta Z_\phi\right)\phi. \tag{175}$$

The relation of two convention is straightforward.

3. δY , δm , δM^2 are also considered to be $O(e^2)$ and δT is $O(e)$.

4. As is mentioned in Sec.3.3, the gauge fixing terms are written in renormalized fields.

5. BRS transformation in Sec.3.2 is defined for the bare fields.

6. Several new vertices which are absent in the tree level appear due to the renormalization.

The relation of bare and renormalized fields are as follows:

1. Gauge fields

$$\begin{gathered}
\underline{W}_\mu^\pm = \sqrt{Z_W} W_\mu^\pm \\
\begin{pmatrix} \underline{Z}_\mu \\ \underline{A}_\mu \end{pmatrix} = \begin{pmatrix} \sqrt{Z}_{AA} & \sqrt{Z}_{ZA} \\ \sqrt{Z}_{AZ} & \sqrt{Z}_{AA} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\
\underline{M}_W^2 = M_W^2 + \delta M_W^2 \\
\underline{M}_Z^2 = M_Z^2 + \delta M_Z^2
\end{gathered} \tag{176}$$

2. Fermions

$$\begin{gathered}
\underline{f}_{L,R} = \sqrt{Z}_{fL,R} f_{L,R}, \quad \underline{\bar{f}}_{L,R} = \sqrt{Z}_{fL,R} \bar{f}_{L,R} \\
\underline{m}_f = m_f + \delta m_f
\end{gathered} \tag{177}$$

3. Scalars

$$\begin{gathered}
\underline{H} = \sqrt{Z}_H H \\
\underline{\chi}^\pm = \sqrt{Z}_\chi \chi^\pm \\
\underline{\chi}_3 = \sqrt{Z}_{\chi 3} \chi_3 \\
\underline{M}_H^2 = M_H^2 + \delta M_H^2
\end{gathered} \tag{178}$$

4. Charge

$$\underline{e} = Ye = (1 + \delta Y)e \tag{179}$$

5. Tadpole

$$\underline{T} = T + \delta T \tag{180}$$

6. Ghost

   There is some freedom for the introduction of renormalization constants for the ghost fields. We use the following convention.

$$\begin{aligned}
&\underline{c}^{\pm} = \tilde{Z}_3 c^{\pm} \\
&\begin{pmatrix} \underline{c}^Z \\ \underline{c}^A \end{pmatrix} = \begin{pmatrix} \tilde{Z}_{AA} & \tilde{Z}_{ZA} \\ \tilde{Z}_{AZ} & \tilde{Z}_{AA} \end{pmatrix} \begin{pmatrix} c^Z \\ c_A \end{pmatrix} \\
&\underline{\bar{c}}^{\pm} = \bar{c}^{\pm} \\
&\underline{\bar{c}}^{Z} = \bar{c}^{Z} \\
&\underline{\bar{c}}^{A} = \bar{c}^{A}
\end{aligned} \tag{181}$$

Following the Kyoto group, we use the following notations.

$$\text{fermion} \qquad \delta G_{mj} = \frac{\delta m_j}{m_j} \tag{182}$$

$$\text{Higgs} \qquad \delta G_H = \frac{\delta M_H^2}{M_H^2} \tag{183}$$

$$\delta G_W = \frac{\delta M_W^2}{2M_W^2} \tag{184}$$

$$\delta G_Z = \frac{\delta M_Z^2}{2M_Z^2} \tag{185}$$

$$\delta H = \frac{\delta M_Z^2 - \delta M_W^2}{2(M_Z^2 - M_W^2)} \tag{186}$$

$$\delta G_1 = \delta G_W - \delta H \tag{187}$$

$$\delta G_2 = \delta G_Z - \delta H \tag{188}$$

$$\delta G_3 = \delta G_Z - \delta G_W \tag{189}$$

$$\delta G_4 = \frac{2\delta M_W^2 - \delta M_Z^2}{2M_W^2 - M_Z^2} - \delta G_W - \delta H \tag{190}$$

These notations are convenient for the description of counter terms. The shift of coupling constants can be written by above δ's. Some examples are shown below.

$$\underline{M}_W = M_W(1 + \delta G_W), \quad \underline{M}_Z = M_Z(1 + \delta G_Z) \tag{191}$$

$$\frac{1}{\underline{c}_W} = \frac{1}{c_W}(1 + \delta G_3) \tag{192}$$

$$\frac{1}{\underline{s}_W} = \frac{1}{s_W}(1 + \delta G_2) \tag{193}$$

$$\frac{\underline{c}_W}{\underline{s}_W} = \frac{c_W}{s_W}(1 + \delta G_1) \tag{194}$$

$$\frac{\underline{c}_W^2 - \underline{s}_W^2}{\underline{s}_W \underline{c}_W} = \frac{c_W^2 - s_W^2}{s_W c_W}(1 + \delta G_4) \tag{195}$$

7.2 Counter terms in one-loop order

The construction of counter terms is done in the followings. The tree Lagrangian in the last sections is written in bare quantities. We replace the bare quantities by the renormalized ones and expand the expression. The terms with δ are counter terms. For the one-loop order, we only keep those up to the first order of δ. Below, examples of the procedure are shown for a generic field ϕ.
Counter terms for a self-energy (propagator)

$$
\begin{aligned}
(\partial_\mu \underline{\phi})^2 - \underline{M}^2 \underline{\phi}^2 &= \left(\partial_\mu \sqrt{Z}_\phi \phi\right)^2 - \left(M^2 + \delta M^2\right) \sqrt{Z}_\phi \phi^2 \\
&= (\partial_\mu \phi)^2 - M^2 \phi^2 + \underbrace{2\delta Z_\phi^{1/2}[(\partial_\mu \phi)^2 - M^2\phi^2] - \delta M^2 \phi^2}_{\text{counter terms}}
\end{aligned} \tag{196}
$$

Counter terms for a vertex

$$
\begin{aligned}
\underline{e}\underline{\phi}_a \underline{\phi}_b \underline{\phi}_c &= Y e \sqrt{Z}_\phi \phi_a \sqrt{Z}_\phi \phi_b \sqrt{Z}_\phi \phi_c \\
&= e\phi_a\phi_b\phi_c + \underbrace{\left(\delta Y + \delta Z_{\phi_\mathrm{a}}^{1/2} + \delta Z_{\phi_\mathrm{b}}^{1/2} + \delta Z_{\phi_\mathrm{c}}^{1/2}\right) e\phi_a\phi_b\phi_c}_{\text{counter terms}}
\end{aligned} \tag{197}
$$

As is seen above, a vertex counter term can be obtained by multiplying a factor to that in the tree level. It is not the case for vertex with A and/or Z since there is a mixing between A and Z:

$$
\underline{A}_\mu = (1 + \delta Z_{AA}^{1/2}) A_\mu + \delta Z_{AZ}^{1/2} Z_\mu, \quad \underline{Z}_\mu = (1 + \delta Z_{ZZ}^{1/2}) Z_\mu + \delta Z_{ZA}^{1/2} A_\mu. \tag{198}
$$

The mixing sometimes produces a counter term which is absent in the tree level. For instance, there is ZZH vertex but ZAH one in the tree level. In the one-loop, the ZAH vertex diagrams exist and its divergence is canceled by the ZAH counter term which is generated by the field mixing.

As was mentioned several times before, the gauge fixing terms are written in renormalized fields, so that they do not generate any counter terms. This means that a counter term which does not include ghost particles is independent of the gauge fixing. In such a counter term, neither ξ's nor $\tilde{\alpha}$ etc. appears. We must be careful for $L^{(2)}(boson)$ in Eq.125 since it is the mixture of bare fields and renormalized fields. The cancellation of vector-scalar transition term (e.g., $M_W \partial_\mu \chi^+ W^{-\mu}$) is not complete and generate counter terms like $-(\delta G_W + \delta Z_W^{1/2} + \delta Z_\chi^{1/2}) M_W (\partial_\mu \chi^+ W^{-\mu})$.

Another point is that we have put $T = 0$ to write down the Feynman rules in the tree level. The tadpole is also shifted as $T + \delta T$ and δT is to be kept when we expand $L(pot)$ in Eq.70 $\sim$ Eq.72.

The word 'counter terms' is sometimes misleading. In the following table, we classify terms in the total Lagrangian after the replacement of bare quantities by renormalized ones.

fields	order	$\delta Z, \delta M$	meaning
quadratic	$O(1)$	no	free part of the Lagrangian define propagators
quadratic	$O(e^n)$ $n \geq 2$	with	counter terms for two-point functions (propagators)
3rd or 4th	$O(e, e^2)$	no	vertices
3rd or 4th	$O(e^n)$ $n \geq 3$	with	vertex counter terms
linear		δT	tadpole counter term
constant			(no meaning)

In the table the first line defines the free Lagrangian, L^{free}, and the rest of all consist of the interaction Lagrangian, L^{int}. The perturbative calculation is done by expanding $\exp[iL^{int}]$, so that the separation of vertex terms and counter terms is not essential. For an instance, when we calculate $O(e^3)$ contribution by perturbation, we receive the contribution from $[L^{int}(O(e)\text{ vertex})]^3$ and $L^{int}(O(e^3)\text{ vertex})$ and the latter is called as 'counter terms'.

7.3 Counter terms in ghost sector

The counter terms including ghost particles are not necessarily in the one-loop order, so that we do not present their explicit formula here. Below, the procedure to determine those counter terms is described briefly.

1. Since gauge fixing terms are given in renormalized quantities, we write the gauge fixing terms by bare ones to fix the bare F functions. From Sec.3.3 and Sec.3.4,

$$\begin{aligned} L(G.F.) = \; & B^+(\partial^\mu W_\mu^- + \xi_W M_W \chi^-) + (h.c.) \\ & + B^Z(\partial^\mu Z_\mu + \xi_Z M_Z \chi_3) + B^A \partial A_\mu \\ & + \text{non}-\text{linear gauge terms} \\ & + B'\text{s bi}-\text{linear terms}. \end{aligned} \tag{199}$$

2. Yet the renormalization of the B fields and ξ's (gauge parameters) is not yet fixed. We can use this freedom to define the bare gauge fixing Lagrangian.

   The renormalization of them is defined so as to erase all Z factors in the bare gauge fixing Lagrangian.

3. Based on the policy stated above, we define the renormalization of B fields as follows.

$$\underline{B}^\pm = \sqrt{Z}_W{}^{-1} B^\pm, \quad \begin{pmatrix} \underline{B}^Z \\ \underline{B}^A \end{pmatrix} = \begin{pmatrix} \sqrt{Z}_{ZZ} & \sqrt{Z}_{ZA} \\ \sqrt{Z}_{AZ} & \sqrt{Z}_{AA} \end{pmatrix}^{-1} \begin{pmatrix} B^Z \\ B^A \end{pmatrix} \tag{200}$$

   The relation is just the inverse of that for gauge fields.

   Then Eq.199 is

$$\begin{aligned} L(G.F.) = \; & \underline{B}^+ \partial^\mu \underline{W}_\mu^- + B^+ \xi_W M_W \chi^- + (h.c.) \\ & + \underline{B}^Z \partial^\mu \underline{Z}_\mu + B^Z \xi_Z M_Z \chi_3 + \underline{B}^A \partial \underline{A}_\mu \\ & + \text{non}-\text{linear gauge terms} \\ & + B'\text{s bi}-\text{linear terms}. \end{aligned} \tag{201}$$

   The remaining terms in Eq.201 become as follows

$$B^+ \xi_W M_W \chi^- = \underline{B}^+ \underline{\xi}_W \underline{M}_W \underline{\chi}^-, \tag{202}$$

$$B^Z \xi_Z M_Z \chi_3 = \underline{B}^Z \underline{\xi}_Z \underline{M}_Z \underline{\chi}_3 + \underline{B}^A \underline{\beta} \underline{M}_Z \underline{\chi}_3 \tag{203}$$

where we defined the renormalization of gauge parameters as

$$\begin{aligned} \underline{\xi}_W &= \xi_W (M_W/\underline{M}_W)\sqrt{Z}_W \sqrt{Z}_\chi^{-1} \\ \underline{\xi}_Z &= \xi_Z (M_Z/\underline{M}_Z)\sqrt{Z}_{ZZ} \sqrt{Z}_{\chi 3}^{-1} \\ \underline{\beta} &= \xi_Z (M_Z/\underline{M}_Z)\sqrt{Z}_{ZA} \sqrt{Z}_{\chi 3}^{-1} \end{aligned} \tag{204}$$

in order to erase all Z. The $\underline{\beta}$ is not an independent parameter but the short-hand notation given by the equation. Non-linear gauge terms can be transformed in a similar way by the renormalization of $\tilde{\alpha}$ etc.

4. By the renormalization of B's and ξ's, the bi-linear terms for B fields, $\xi B^+ B^- + \cdots$, receive extra factors. However this does not affects the renormalization program since $\delta_B B = 0$.

5. We obtain bare F terms by the above equations.

$$\begin{aligned} \underline{F}^{\mp} &= \partial^\mu \underline{W}_\mu^{\mp} + \underline{\xi}_W \underline{M}_W \underline{\chi}^{\mp} + \mathrm{non-linear\ gauge\ terms} \\ \underline{F}^{Z} &= \partial^\mu \underline{Z}_\mu + \underline{\xi}_Z \underline{M}_Z \underline{\chi}_3 + \mathrm{non-linear\ gauge\ terms} \\ \underline{F}^{A} &= \partial^\mu \underline{A}_\mu + \underline{\beta} \underline{M}_Z \underline{\chi}_3 + \mathrm{non-linear\ gauge\ terms} \end{aligned} \tag{205}$$

Except for F^A, these are the same as those in Eq.134 assuming that quantities are bare ones. In Eq.134 $F^A = \partial^\mu A_\mu$, while we have additional terms here.

6. The BRS transformation is defined for the bare fields. The ghost Lagrangian is $L(F.P.) = i\underline{\bar{c}}\delta_B \underline{F}^- + \cdots$ and the application of BRS transformation gives the explicit form as in Sec.3.6. Since F^A differs, we have additional terms which are absent in Sec.3.6 (and also in Sec.6). However, $\underline{\beta}$ itself is $O(e^2)$ quantity and it generates $O(e^3)$ vertices. Thus the appearance of $\underline{\beta}$ is only in the counter terms in the one-loop calculation.

7. We have obtained the $L(F.P.)$ expressed by the bare quantities. Then the bare quantities are replaced by the renormalized ones. The separation of the counter terms can be done as before.

7.4 Counter terms for vertices

Here, we summarize the vertex counter terms. Ghost vertices are not shown since they are not necessary in the one-loop order. Counter terms for the propagators and the tadpole are discussed in the next subsection. In this subsection $\langle \cdots \rangle$ stands for the denoted tree vertex defined in Sec.6 after taking $\tilde{\alpha} = \tilde{\beta} = \tilde{\delta} = \tilde{\varepsilon} = \tilde{\kappa} = 0$. The vertex which is absent in the tree level is denoted by (new).

The following counter terms can be obtained easily by inspection of tree Feynman rules in Sec.6 and the operation demonstrated in Sec.7.2. However, as we put $T = 0$ to obtain the tree rules, the appearance of δT should be traced back to $L(pot)$.

7.4.1 Vector-Vector-Vector

p_1 (μ)	p_2 (ν)	p_3 (ρ)	
W^-	W^+	A	$(\delta Y + 2\delta Z_W^{1/2} + \delta Z_{AA}^{1/2})\langle WWA\rangle + \delta Z_{ZA}^{1/2}\langle WWZ\rangle$
W^-	W^+	Z	$(\delta Y + \delta G_1 + 2\delta Z_W^{1/2} + \delta Z_{ZZ}^{1/2})\langle WWZ\rangle + \delta Z_{AZ}^{1/2}\langle WWA\rangle$

7.4.2 Vector-Vector-Scalar

p_1 (μ)	p_2 (ν)	p_3	
$W^\pm$	A	$\chi^\mp$	$(\delta Y + \delta G_W + \delta Z_W^{1/2} + \delta Z_\chi^{1/2} + \delta Z_{AA}^{1/2})\langle WA\chi\rangle + \delta Z_{ZA}^{1/2}\langle WZ\chi\rangle$
$W^\pm$	Z	$\chi^\mp$	$(\delta Y + \delta H + \delta Z_W^{1/2} + \delta Z_\chi^{1/2} + \delta Z_{ZZ}^{1/2})\langle WZ\chi\rangle + \delta Z_{AZ}^{1/2}\langle WA\chi\rangle$
W^-	W^+	H	$(\delta Y + \delta G_2 + \delta G_W + 2\delta Z_W^{1/2} + \delta Z_H^{1/2})\langle WWH\rangle$
Z	Z	H	$(\delta Y + \delta G_2 + \delta G_3 + \delta G_Z + 2\delta Z_{ZZ}^{1/2} + \delta Z_H^{1/2})\langle ZZH\rangle$
Z	A	H	$\delta Z_{ZA}^{1/2}\langle ZZH\rangle$ (new)

7.4.3 Scalar-Scalar-Vector

p_1	p_2	p_3 (μ)	
H	$\chi^\mp$	$W^\pm$	$(\delta Y + \delta G_2 + \delta Z_H^{1/2} + \delta Z_\chi^{1/2} + \delta Z_W^{1/2})\langle H\chi W\rangle$
χ_3	$\chi^\mp$	$W^\pm$	$(\delta Y + \delta G_2 + \delta Z_{\chi 3}^{1/2} + \delta Z_\chi^{1/2} + \delta Z_W^{1/2})\langle \chi_3\chi W\rangle$
χ^-	χ^+	A	$(\delta Y + 2\delta Z_\chi^{1/2} + \delta Z_{AA}^{1/2})\langle\chi\chi A\rangle + \delta Z_{ZA}^{1/2}\langle\chi\chi Z\rangle$
χ^-	χ^+	Z	$(\delta Y + \delta G_4 + 2\delta Z_\chi^{1/2} + \delta Z_{ZZ}^{1/2})\langle\chi\chi Z\rangle + \delta Z_{AZ}^{1/2}\langle\chi\chi A\rangle$
H	χ_3	Z	$(\delta Y + \delta G_2 + \delta G_3 + \delta Z_H^{1/2} + \delta Z_{\chi 3}^{1/2} + \delta Z_{ZZ}^{1/2})\langle H\chi_3 Z\rangle$
H	χ_3	A	$\delta Z_{ZA}^{1/2}\langle H\chi_3 Z\rangle$ (new)

7.4.4 Scalar-Scalar-Scalar

(See Eq.71.)

p_1	p_2	p_3	
H	H	H	$\left[(\delta Y + \delta G_2 - \delta G_W + \delta G_H + 3\delta Z_H^{1/2}) - \delta T\frac{e}{s_W M_W M_H^2}\right]\langle HHH\rangle$
H	χ^-	χ^+	$\left[(\delta Y + \delta G_2 - \delta G_W + \delta G_H + \delta Z_H^{1/2} + 2\delta Z_\chi^{1/2}) - \delta T\frac{e}{s_W M_W M_H^2}\right]\langle H\chi\chi\rangle$
H	χ_3	χ_3	$\left[(\delta Y + \delta G_2 - \delta G_W + \delta G_H + \delta Z_H^{1/2} + 2\delta Z_{\chi 3}^{1/2}) - \delta T\frac{e}{s_W M_W M_H^2}\right]\langle H\chi_3\chi_3\rangle$

7.4.5 Vector-Vector-Vector-Vector

p_1 (μ)	p_2 (ν)	p_3 (ρ)	p_4 (σ)	
W^+	W^-	A	A	$(2\delta Y + 2\delta Z_W^{1/2} + 2\delta Z_{AA}^{1/2})\langle WWAA\rangle$ $+2\delta Z_{ZA}^{1/2}\langle WWAZ\rangle$
W^+	W^-	A	Z	$(2\delta Y + \delta G_1 + 2\delta Z_W^{1/2} + \delta Z_{AA}^{1/2} + \delta Z_{ZZ}^{1/2})\langle WWAZ\rangle$ $+\delta Z_{AZ}^{1/2}\langle WWAA\rangle + \delta Z_{ZA}^{1/2}\langle WWZZ\rangle$
W^+	W^-	Z	Z	$(2\delta Y + 2\delta G_1 + 2\delta Z_W^{1/2} + 2\delta Z_{ZZ}^{1/2})\langle WWZZ\rangle$ $+2\delta Z_{AZ}^{1/2}\langle WWAZ\rangle$
W^+	W^-	W^-	W^+	$(2\delta Y + 2\delta G_2 + 4\delta Z_W^{1/2})\langle WWWW\rangle$

7.4.6 Vector-Vector-Scalar-Scalar

p_1 (μ)	p_2 (ν)	p_3	p_4	
A	$W^\pm$	H	$\chi^\mp$	$(2\delta Y+\delta G_2+\delta Z_{AA}^{1/2}+\delta Z_W^{1/2}+\delta Z_H^{1/2}+\delta Z_\chi^{1/2})\langle AWH\chi\rangle$ $+\delta Z_{ZA}^{1/2}\langle ZWH\chi\rangle$
A	$W^\pm$	χ_3	$\chi^\mp$	$(2\delta Y+\delta G_2+\delta Z_{AA}^{1/2}+\delta Z_W^{1/2}+\delta Z_{\chi 3}^{1/2}+\delta Z_\chi^{1/2})\langle AW\chi_3\chi\rangle$ $+\delta Z_{ZA}^{1/2}\langle ZW\chi_3\chi\rangle$
Z	$W^\pm$	H	$\chi^\mp$	$(2\delta Y+\delta G_3+\delta Z_{ZZ}^{1/2}+\delta Z_W^{1/2}+\delta Z_H^{1/2}+\delta Z_\chi^{1/2})\langle ZWH\chi\rangle$ $+\delta Z_{AZ}^{1/2}\langle AWH\chi\rangle$
Z	$W^\pm$	χ_3	$\chi^\mp$	$(2\delta Y+\delta G_3+\delta Z_{ZZ}^{1/2}+\delta Z_W^{1/2}+\delta Z_{\chi 3}^{1/2}+\delta Z_\chi^{1/2})\langle ZW\chi_3\chi\rangle$ $+\delta Z_{AZ}^{1/2}\langle AW\chi_3\chi\rangle$
A	A	χ^+	χ^-	$(2\delta Y+2\delta Z_{AA}^{1/2}+2\delta Z_\chi^{1/2})\langle AA\chi\chi\rangle+2\delta Z_{ZA}^{1/2}\langle ZA\chi\chi\rangle$
Z	A	χ^+	χ^-	$(2\delta Y+\delta G_4+\delta Z_{ZZ}^{1/2}+\delta Z_{AA}^{1/2}+2\delta Z_\chi^{1/2})\langle ZA\chi\chi\rangle$ $+\delta Z_{ZA}^{1/2}\langle ZZ\chi\chi\rangle+\delta Z_{AZ}^{1/2}\langle AA\chi\chi\rangle$
Z	Z	χ^+	χ^-	$(2\delta Y+2\delta G_4+2\delta Z_{ZZ}^{1/2}+2\delta Z_\chi^{1/2})\langle ZZ\chi\chi\rangle$ $+2\delta Z_{AZ}^{1/2}\langle ZA\chi\chi\rangle$
W^+	W^-	H	H	$(2\delta Y+2\delta G_2+2\delta Z_W^{1/2}+2\delta Z_H^{1/2})\langle WWHH\rangle$
W^+	W^-	χ_3	χ_3	$(2\delta Y+2\delta G_2+2\delta Z_W^{1/2}+2\delta Z_{\chi 3}^{1/2})\langle WW\chi_3\chi_3\rangle$
W^+	W^-	χ^-	χ^+	$(2\delta Y+2\delta G_2+2\delta Z_W^{1/2}+2\delta Z_\chi^{1/2})\langle WW\chi\chi\rangle$
Z	Z	H	H	$(2\delta Y+2\delta G_2+2\delta G_3+2\delta Z_{ZZ}^{1/2}+2\delta Z_H^{1/2})\langle ZZHH\rangle$
Z	Z	χ_3	χ_3	$(2\delta Y+2\delta G_2+2\delta G_3+2\delta Z_{ZZ}^{1/2}+2\delta Z_{\chi 3}^{1/2})\langle ZZ\chi_3\chi_3\rangle$
Z	A	H	H	$\delta Z_{ZA}^{1/2}\langle ZZHH\rangle$ (new)
Z	A	χ_3	χ_3	$\delta Z_{ZA}^{1/2}\langle ZZ\chi_3\chi_3\rangle$ (new)

7.4.7 Scalar-Scalar-Scalar-Scalar

(See Eq.72.)

p_1	p_2	p_3	p_4	
H	H	H	H	$\Big[(2\delta Y+2\delta G_2-2\delta G_W+\delta G_H+4\delta Z_H^{1/2})$ $-\delta T\frac{e}{s_W M_W M_H^2}\Big]\langle HHHH\rangle$
χ_3	χ_3	χ_3	χ_3	$\Big[(2\delta Y+2\delta G_2-2\delta G_W+\delta G_H+4\delta Z_{\chi 3}^{1/2})$ $-\delta T\frac{e}{s_W M_W M_H^2}\Big]\langle \chi_3\chi_3\chi_3\chi_3\rangle$
$\chi^\pm$	$\chi^\mp$	$\chi^\mp$	$\chi^\pm$	$\Big[(2\delta Y+2\delta G_2-2\delta G_W+\delta G_H+4\delta Z_\chi^{1/2})$ $-\delta T\frac{e}{s_W M_W M_H^2}\Big]\langle \chi\chi\chi\chi\rangle$
H	H	χ_3	χ_3	$\Big[(2\delta Y+2\delta G_2-2\delta G_W+\delta G_H+2\delta Z_H^{1/2}+2\delta Z_{\chi 3}^{1/2})$ $-\delta T\frac{e}{s_W M_W M_H^2}\Big]\langle HH\chi_3\chi_3\rangle$
H	H	χ^+	χ^-	$\Big[(2\delta Y+2\delta G_2-2\delta G_W+\delta G_H+2\delta Z_H^{1/2}+2\delta Z_\chi^{1/2})$ $-\delta T\frac{e}{s_W M_W M_H^2}\Big]\langle HH\chi\chi\rangle$
χ^+	χ^-	χ_3	χ_3	$\Big[(2\delta Y+2\delta G_2-2\delta G_W+\delta G_H+2\delta Z_\chi^{1/2}+2\delta Z_{\chi 3}^{1/2})$ $-\delta T\frac{e}{s_W M_W M_H^2}\Big]\langle \chi\chi\chi_3\chi_3\rangle$

7.4.8 Fermion-Fermion-Vector

$L, R = (1 \mp \gamma_5)/2$

p_1	p_2	p_3 (μ)	
$\bar{f}$	f	A	$(\delta Y + \delta Z^{1/2}_{AA} + \delta Z^{1/2}_{fL} + \delta Z^{1/2}_{fL}) e Q_f \gamma^\mu L$
			$+(\delta Y + \delta Z^{1/2}_{AA} + \delta Z^{1/2}_{fR} + \delta Z^{1/2}_{fR}) e Q_f \gamma^\mu R$
			$+\delta Z^{1/2}_{ZA} \frac{e}{2 s_W c_W} \Big(2 I_3 \gamma^\mu L - 2 s_W^2 Q_f \gamma^\mu (L+R) \Big)$
$\bar{f}$	f	Z	$(\delta Y + \delta G_2 + \delta G_3 + \delta Z^{1/2}_{ZZ} + \delta Z^{1/2}_{fL} + \delta Z^{1/2}_{fL}) \frac{e}{2 s_W c_W} 2 I_3 \gamma^\mu L$
			$+(\delta Y - \delta G_2 + \delta G_3 + \delta Z^{1/2}_{ZZ} + \delta Z^{1/2}_{fL} + \delta Z^{1/2}_{fL}) \frac{e}{2 s_W c_W} (-2 s_W^2 Q_f \gamma^\mu L)$
			$+(\delta Y - \delta G_2 + \delta G_3 + \delta Z^{1/2}_{ZZ} + \delta Z^{1/2}_{fR} + \delta Z^{1/2}_{fR}) \frac{e}{2 s_W c_W} (-2 s_W^2 Q_f \gamma^\mu R)$
			$+\delta Z^{1/2}_{AZ} e Q_f \gamma^\mu (L+R)$
$\bar{U}/\bar{D}$	D/U	W^+/W^-	$(\delta Y + \delta G_2 + \delta Z^{1/2}_{(U/D)L} + \delta Z^{1/2}_{(D/U)L} + \delta Z^{1/2}_{W}) \frac{e}{\sqrt{2} s_W} \gamma^\mu L$

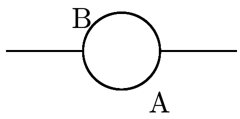
7.4.9 Fermion-Fermion-Scalar

$L, R = (1 \mp \gamma_5)/2$

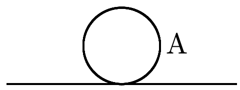
p_1	p_2	p_3	
$\bar{f}$	f	H	
			$(\delta Y + \delta G_2 + \delta G_{mf} - \delta G_W + \delta Z^{1/2}_{fR} + \delta Z^{1/2}_{fL} + \delta Z^{1/2}_{H}) \left(-\frac{e}{2 s_W} \frac{m_f}{M_W} \right) L$
			$+(\delta Y + \delta G_2 + \delta G_{mf} - \delta G_W + \delta Z^{1/2}_{fL} + \delta Z^{1/2}_{fR} + \delta Z^{1/2}_{H}) \left(-\frac{e}{2 s_W} \frac{m_f}{M_W} \right) R$
$\bar{U}/\bar{D}$	U/D	χ_3	
			$(\delta Y + \delta G_2 + \delta G_{mf} - \delta G_W + \delta Z^{1/2}_{(U/D)R} + \delta Z^{1/2}_{(U/D)L} + \delta Z^{1/2}_{\chi 3}) \left((-/+) \frac{ie}{2 s_W} \frac{m_f}{M_W} \right) (-L)$
			$+(\delta Y + \delta G_2 + \delta G_{mf} - \delta G_W + \delta Z^{1/2}_{(U/D)L} + \delta Z^{1/2}_{(U/D)R} + \delta Z^{1/2}_{\chi 3}) \left((-/+) \frac{ie}{2 s_W} \frac{m_f}{M_W} \right) R$
$\bar{U}$	D	χ^+	$(\delta Y + \delta G_2 + \delta G_{mU} - \delta G_W + \delta Z^{1/2}_{U,R} + \delta Z^{1/2}_{D,L} + \delta Z^{1/2}_{\chi}) \frac{-ie}{\sqrt{2} s_W} \frac{m_U}{M_W} (-L)$
			$+(\delta Y + \delta G_2 + \delta G_{mD} - \delta G_W + \delta Z^{1/2}_{U,L} + \delta Z^{1/2}_{D,R} + \delta Z^{1/2}_{\chi}) \frac{-ie}{\sqrt{2} s_W} \frac{m_D}{M_W} R$
$\bar{D}$	U	χ^-	$(\delta Y + \delta G_2 + \delta G_{mD} - \delta G_W + \delta Z^{1/2}_{D,R} + \delta Z^{1/2}_{U,L} + \delta Z^{1/2}_{\chi}) \frac{-ie}{\sqrt{2} s_W} \frac{m_D}{M_W} (-L)$
			$+(\delta Y + \delta G_2 + \delta G_{mU} - \delta G_W + \delta Z^{1/2}_{D,L} + \delta Z^{1/2}_{U,R} + \delta Z^{1/2}_{\chi}) \frac{-ie}{\sqrt{2} s_W} \frac{m_U}{M_W} R$

7.5 Renormalization conditions

In Sec.7.1, we have introduced renormalization constants, e.g., $\sqrt{Z}$, δM^2 and so forth. By these constants, counter terms are described. These constants are determined by the renormalization conditions described below. There are many possible way to present the conditions. We follow the on-mass-shell renormalization scheme given by Kyoto group.



(a) two $O(e)$ vertices



(b) one $O(e^2)$ vertex

(c) $O(e^2)$ vertex (counter term)

Figure 3: Two point functions

In the $O(e^2)$ perturbation, we have the contribution for the two point functions shown in Fig.3. The contribution from (a) and (b) is called the loop term, and that from (c) the counter term. When we denote the former as Π, the latter is denoted by $\hat{\Pi}$. The sum of these is represented as $\tilde{\Pi} = \Pi + \hat{\Pi}$.

The decomposition of two point functions are as follows.

type	formula
vector-vector	$\Pi_{\mu\nu}(q^2) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_T(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_L(q^2)$
scalar-scalar	$\Pi(q^2)$
vector-scalar	$iq_\mu\Pi(q^2)$ (q flows in from scalar)
fermion-fermion	$\Sigma(q^2) = K_1 I + K_5\gamma_5 + K_\gamma \not{q} + K_{5\gamma} \not{q}\gamma_5$

The counter terms are given as follows.

1. Vector-Vector

WW	$\hat{\Pi}_T^W = \delta M_W^2 + 2(M_W^2 - q^2)\delta Z_W^{1/2}$
	$\hat{\Pi}_L^W = \delta M_W^2 + 2M_W^2\delta Z_W^{1/2}$
ZZ	$\hat{\Pi}_T^{ZZ} = \delta M_Z^2 + 2(M_Z^2 - q^2)\delta Z_{ZZ}^{1/2}$
	$\hat{\Pi}_L^{ZZ} = \delta M_Z^2 + 2M_Z^2\delta Z_{ZZ}^{1/2}$
ZA	$\hat{\Pi}_T^{ZA} = (M_Z^2 - q^2)\delta Z_{ZA}^{1/2} - q^2\delta Z_{AZ}^{1/2}$
	$\hat{\Pi}_L^{ZA} = M_Z^2\delta Z_{ZA}^{1/2}$
AA	$\hat{\Pi}_T^{AA} = -2q^2\delta Z_{AA}^{1/2}$
	$\hat{\Pi}_L^{AA} = 0$

2. Scalar-Scalar

They originate from $L^{(2)}(boson)$ in Eq.125 including δT terms from the last line of the equation.

HH	$\hat{\Pi}^H = 2(q^2 - M_H^2)\delta Z_H^{1/2} - \delta M_H^2 + \frac{3\delta T}{v}$
$\chi_3\chi_3$	$\hat{\Pi}^{\chi_3} = 2q^2\delta Z_{\chi_3}^{1/2} + \frac{\delta T}{v}$
$\chi\chi$	$\hat{\Pi}^{\chi} = 2q^2\delta Z_{\chi}^{1/2} + \frac{\delta T}{v}$

3. Vector-Scalar

$W\chi$	$\hat{\Pi}^{W\chi} = M_W(\delta G_W + \delta Z_W^{1/2} + \delta Z_\chi^{1/2})$
$Z\chi_3$	$\hat{\Pi}^{Z\chi_3} = M_Z(\delta G_Z + \delta Z_{ZZ}^{1/2} + \delta Z_{\chi_3}^{1/2})$
$A\chi_3$	$\hat{\Pi}^{A\chi_3} = M_Z\delta Z_{ZA}^{1/2}$

4. Fermion-Fermion

The L^{free} for the fermion f is $\bar{f}(i\gamma^\mu\partial_\mu - m_f)f$. It is

$$\begin{array}{l} \bar{f}_L(i\gamma^\mu\partial_\mu)Lf_L \\ +\bar{f}_R(i\gamma^\mu\partial_\mu)Rf_R \\ -\bar{f}_R m_f L f_L \\ -\bar{f}_L m_f R f_R \end{array} \tag{206}$$

and it gives the counter terms:

$$\begin{array}{l} 2\delta Z_{fL}^{1/2}\bar{f}_L(i\gamma^\mu\partial_\mu)Lf_L \\ +2\delta Z_{fR}^{1/2}\bar{f}_R(i\gamma^\mu\partial_\mu)Rf_R \\ -(\delta m_f + (\delta Z_{fR}^{1/2} + \delta Z_{fL}^{1/2})m_f)\bar{f}_R L f_L \\ -(\delta m_f + (\delta Z_{fL}^{1/2} + \delta Z_{fH}^{1/2})m_f)\bar{f}_L R f_R \end{array} \tag{207}$$

Rearrangement of the above gives the following terms.

ff	$\hat{K}_1 = -m_f\left(\delta Z_{fL}^{1/2} + \delta Z_{fR}^{1/2}\right) - \delta m_f$
	$\hat{K}_5 = +\frac{1}{2}m_f\left(-\delta Z_{fL}^{1/2} + \delta Z_{fL}^{1/2} + \delta Z_{fR}^{1/2} - \delta Z_{fR}^{1/2}\right) = 0$
	$\hat{K}_\gamma = +\delta Z_{fL}^{1/2} + \delta Z_{fR}^{1/2}$
	$\hat{K}_{5\gamma} = -\delta Z_{fL}^{1/2} + \delta Z_{fR}^{1/2}$

For the Higgs one-point function, we have the contribution shown in Fig.4. The loop term (a) is T^{loop} and the counter term (b) is δT. The sum is $\tilde{T} = T^{loop} + \delta T$.

Now, we introduce the renormalization conditions. Those for ghost particles are skipped by the same reason as before.

(a) One $O(e)$ vertex (b) δT

Figure 4: One point functions

1. Tadpole

   This is to cancel the tadpole.

$$\tilde{T} = 0 \tag{208}$$

   Then

$$\delta T = -T^{loop}. \tag{209}$$

2. Charged vector

   The conditions specify that the pole-position of the propagator is M_W^2, and that the residue of the propagator at the pole is 1.

$$\tilde{\Pi}_T^W(M_W^2) = 0, \quad \frac{d}{dq^2}\tilde{\Pi}_T^W(M_W^2) = 0 \tag{210}$$

   This gives the following relations.

$$\delta M_W^2 = -\Pi_T^W(M_W^2), \quad \delta Z_W^{1/2} = \frac{1}{2}\frac{d}{dq^2}\Pi_T^W(M_W^2) \tag{211}$$

3. Neutral vector

   The conditions same as W is given for pole-positions and residues of both Z and A. Also we have the condition that there is no mixing between Z and A at $q^2 = 0, M_Z^2$.

$$\tilde{\Pi}_T^{ZZ}(M_Z^2) = 0, \quad \frac{d}{dq^2}\tilde{\Pi}_T^{ZZ}(M_Z^2) = 0 \tag{212}$$

$$\tilde{\Pi}_T^{AA}(0) = 0, \quad \frac{d}{dq^2}\tilde{\Pi}_T^{AA}(0) = 0 \tag{213}$$

$$\tilde{\Pi}_T^{ZA}(0) = 0, \quad \tilde{\Pi}_T^{ZA}(M_Z^2) = 0 \tag{214}$$

   Among the 6 conditions, $\tilde{\Pi}_T^{AA}(0) = 0$ produces nothing (except for the check of loop calculation to show $\Pi_T^{AA}(0) = 0$). These give the following relations.

$$\delta M_Z^2 = -\Pi_T^{ZZ}(M_Z^2), \quad \delta Z_{ZZ}^{1/2} = \frac{1}{2}\frac{d}{dq^2}\Pi_T^{ZZ}(M_Z^2) \tag{215}$$

$$\delta Z_{AA}^{1/2} = \frac{1}{2}\frac{d}{dq^2}\Pi_T^{AA}(0) \tag{216}$$

$$\delta Z_{ZA}^{1/2} = -\Pi_T^{ZA}(0)/M_Z^2, \quad \delta Z_{AZ}^{1/2} = \Pi_T^{ZA}(M_Z^2)/M_Z^2 \tag{217}$$

4. Higgs

The conditions specify that the pole-position of the propagator is M_H^2, and that the residue of the propagator at the pole is 1.

$$\tilde{\Pi}^H(M_H^2) = 0, \quad \frac{d}{dq^2}\tilde{\Pi}^H(M_H^2) = 0 \tag{218}$$

This gives the following relations.

$$\delta M_H^2 = \Pi^H(M_H^2) + \frac{3\delta T}{v}, \quad \delta Z_H^{1/2} = -\frac{1}{2}\frac{d}{dq^2}\Pi^H(M_H^2) \tag{219}$$

5. χ, χ_3

There is no physical condition for unphysical scalars. However, we are to define $\sqrt{Z}_\chi$ and $\sqrt{Z}_{\chi 3}$. Since the counter term is $\sim 2q^2\sqrt{Z}_\chi$, the so-called $\overline{\text{MS}}$ scheme is used in the linear gauge case:

$$\sqrt{Z}_\chi = -\frac{1}{2q^2}\,\Pi^\chi(q^2)\Big|_{C_{\mathsf{UV}}\ \mathsf{part}}, \qquad \sqrt{Z}_{\chi 3} = -\frac{1}{2q^2}\,\Pi^{\chi 3}(q^2)\Big|_{C_{\mathsf{UV}}\ \mathsf{part}} \tag{220}$$

Thus the counter term is defined just to erase the divergence. In the linear gauge case, the C_{UV} part is proportional to q^2 as

$$\Pi^\chi(q^2) = cq^2 C_{\mathsf{UV}} + (\text{finite terms})$$

so that Eq.220 works well. In the non-linear gauge, the C_{UV} part is no more proportional to q^2, though we still keep the above definition as working hypothesis.

6. Fermion

The conditions for pole-positions and residues are the same as other physical particles. Also the vanishing of γ_5 and $\gamma^\mu\gamma_5$ terms at the pole is required.

$$m_f\tilde{K}_\gamma(m_f^2) + \tilde{K}_1(m_f^2) = 0, \quad \frac{d}{d\not{q}}\left(\not{q}\tilde{K}_\gamma(q^2) + \tilde{K}_1(q^2)\right)\Big|_{\not{q}=m_f} = 0, \tag{221}$$

$$\tilde{K}_5(m_f^2) = 0, \quad \tilde{K}_{5\gamma}(m_f^2) = 0. \tag{222}$$

From the condition for $\tilde{K}_5$, we obtain

$$2K_5(m_f^2) = (\delta Z_{fL}^{1/2} - \delta Z_{fL}^{1/2*}) - (\delta Z_{fR}^{1/2} - \delta Z_{fR}^{1/2*}) = 0. \tag{223}$$

When we respect the CP invariance, $K_5 = 0$ holds. In this case, we can define that $\delta Z_{fL}^{1/2}$ and $\delta Z_{fR}^{1/2}$ are both real using the freedom of phase of the field. Under this situation, we obtain the following relations.

$$\begin{aligned}
\delta m_f &= m_f K_\gamma(m_f^2) + K_1(m_f^2) \\
\delta Z_{fL}^{1/2} &= \tfrac{1}{2}(K_{5\gamma}(m_f^2) - K_\gamma(m_f^2)) - m_f^2\tfrac{d}{dq^2}K_\gamma(m_f^2) - m_f\tfrac{d}{dq^2}K_1(m_f^2) \\
\delta Z_{fR}^{1/2} &= -\tfrac{1}{2}(K_{5\gamma}(m_f^2) + K_\gamma(m_f^2)) - m_f^2\tfrac{d}{dq^2}K_\gamma(m_f^2) - m_f\tfrac{d}{dq^2}K_1(m_f^2)
\end{aligned} \tag{224}$$

7. Charge

   While there are many vertices in the theory, if the charge e is properly renormalized, we do not need any further renormalization conditions.

   The condition can be imposed to any vertex. The most natural one is to fix the eeA vertex. The counter term is already defined in Sec.7.4 and the condition requests that the coupling is $-e$ when q, the momentum of photon, is 0.

$$(eeA \text{ one loop term} + eeA \text{ counter term})\big|_{q=0} = 0 \tag{225}$$

   From this, we obtain δY.

8 One-point functions

The one-loop diagram for (a) in Fig.4 is integrated by the following formula.

$$\int \frac{d^n \ell}{i(2\pi)^n} \frac{1}{\ell^2 - m_A^2} = \frac{1}{16\pi^2} m_A^2 \left(C_{\mathsf{UV}} - \log m_A^2 + 1 \right) \tag{226}$$

Calculation is done by the dimensional regularization with $n = 4 - 2\varepsilon$. Here and in the followings we use the notation,

$$C_{\mathsf{UV}} = \frac{1}{\varepsilon} - \gamma_E + \log 4\pi. \tag{227}$$

In the followings, the summation over fermions $\left(\sum_f, \sum_{doublet} \right)$ implicitly includes the sum over color for quarks.

The T^{loop} is calculated by the diagram (a) in Fig.4 where $A = W, Z, \chi, \chi_3, H, c, c^Z, f$. The result is as follows:

$$\begin{aligned} T^{loop} = \quad & \frac{e}{16\pi^2 s_W M_W} \Big[(C_{\mathsf{UV}} - \log M_W^2 + 1) M_W^2 (3M_W^2 + \frac{1}{2} M_H^2) - 2(M_W^2)^2 \\ & + (C_{\mathsf{UV}} - \log M_Z^2 + 1) M_Z^2 (\frac{3}{2} M_Z^2 + \frac{1}{4} M_H^2) - (M_Z^2)^2 \\ & + (C_{\mathsf{UV}} - \log M_H^2 + 1) \frac{3}{4} (M_H^2)^2 \\ & - \sum_f 2 m_f^4 (C_{\mathsf{UV}} - \log m_f^2 + 1) \Big] \end{aligned} \tag{228}$$

Some of diagrams depend on $\tilde{\varepsilon}$ or $\tilde{\delta}$. However the dependence on non-linear gauge parameters is canceled between scalar loops and ghost loops, so that δT is the same as in the linear gauge.

The tadpole counter term is simply obtained by this formula.

$$\frac{\delta T}{v} = \frac{\alpha}{8\pi s_W^2} \left[C^T C_{\mathsf{UV}} + d_0^T + \sum_f \frac{2m_f^4}{M_W^2} (C_{\mathsf{UV}} + 1 - \log m_f^2) \right] \tag{229}$$

$$C^T = -3M_W^2 - \frac{3}{2}\frac{M_Z^2}{c_W^2} - \left(\frac{1}{2} + \frac{1}{4c_W^2} \right) M_H^2 - \frac{3}{4}\frac{(M_H^2)^2}{M_W^2} \tag{230}$$

$$
\begin{aligned}
d_0^T = & \ (\log M_W^2 - 1)(3M_W^2 + \frac{1}{2}M_H^2) + 2M_W^2 \\
& + \frac{1}{c_W^2}(\log M_Z^2 - 1)(\frac{3}{2}M_Z^2 + \frac{1}{4}M_H^2) + \frac{M_Z^2}{c_W^2} + \frac{3}{4}\frac{(M_H^2)^2}{M_W^2}(\log M_H^2 - 1)
\end{aligned}
\tag{231}
$$

9 Two-point functions

In this section f' stands for the partner fermion in the weak doublet. If $f = U(D)$, $f' = D(U)$.

9.1 Integrals

In the calculation of two-point functions, we deal with the integral for the diagrams in Fig.5. The integral for (b) is the same as in the last section for one-point functions(Eq.226).

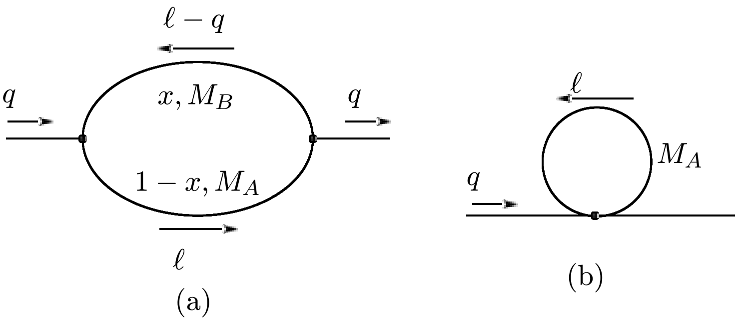


Figure 5: Loop integral for two point functions

The integral for the diagram (a) becomes

$$
\begin{aligned}
& \int \frac{d^n\ell}{i(2\pi)^n} \frac{N}{(\ell^2 - M_A^2)((\ell - q)^2 - M_B^2)} \\
& = \int \frac{d^n\ell}{i(2\pi)^n} \int_0^1 \frac{N}{[(1-x)(\ell^2 - M_A^2) + x((\ell - q)^2 - M_B^2)]^2} dx.
\end{aligned}
\tag{232}
$$

Then the shift of momentum ℓ is done by

$$
\ell \ \Rightarrow \ \ell + xq \tag{233}
$$

in the numerator N and the terms in odd power of ℓ are discarded. The contraction of loop momentum should be done by

$$
\ell^\mu \ell^\nu = \frac{1}{4}\ell^2 g^{\mu\nu} \cdot \left(1 + \frac{\varepsilon}{2}\right). \tag{234}
$$

The integral by the loop momentum can be done by the following formulas:

$$
D_2 = (1-x)M_A^2 + xM_B^2 - x(1-x)s, \quad (s = q^2) \tag{235}
$$

$$
\int \frac{d^n\ell}{i(2\pi)^n} \frac{1}{(\ell^2 - D_2)^2} = \frac{1}{16\pi^2}\left(C_{\mathsf{UV}} - \log D_2\right) \tag{236}
$$

$$\int \frac{d^n\ell}{i(2\pi)^n}\frac{\ell^2}{(\ell^2-D_2)^2} = \frac{1}{16\pi^2}2D_2\left(C_{\mathsf{UV}}+\frac{1}{2}-\log D_2\right) \tag{237}$$

$$\int \frac{d^n\ell}{i(2\pi)^n}\frac{\ell^\mu\ell^\nu}{(\ell^2-D_2)^2} = \frac{1}{16\pi^2}\frac{D_2}{2}\left(C_{\mathsf{UV}}+1-\log D_2\right)g^{\mu\nu} \tag{238}$$

The integral by the parameter x is done by the following formulas:

$$\int_0^1 D_2 dx = \frac{1}{2}(M_A^2+M_B^2)-\frac{1}{6}s \tag{239}$$

$$F_n(A,B) = \int_0^1 x^n \log D_2 dx = \int_0^1 x^n \log\left[(1-x)M_A^2+xM_B^2-x(1-x)s\right]dx \tag{240}$$

The integral of F_n is elementary though its explicit form is not shown here. We only encounter the integrals up to $n=2$. The notation $F_{12}(A,B) = F_1(A,B)-F_2(A,B) = F_{12}(B,A)$ is sometimes used.

$$\begin{aligned}\tilde{F}(A,B) = & \int_0^1 D_2 \log D_2 dx \\ & = M_A^2\left(F_0(A,B)-F_1(A,B)\right)+M_B^2F_1(A,B)-sF_{12}(A,B)\end{aligned} \tag{241}$$

We have several relations for F_n functions as shown below. By use of reduction rules, we can convert all F_n's into F_0.

Exchange of A and B

$$\begin{aligned}F_0(B,A) =& \; F_0(A,B)\\ F_1(B,A) =& \; F_0(A,B)-F_1(A,B)\\ F_2(B,A) =& \; F_0(A,B)-2F_1(A,B)+F_2(A,B)\end{aligned} \tag{242}$$

Reduction into F_0, $A\neq B$

$$F_1(A,B) = \frac{1}{2}\left(1+\frac{M_A^2-M_B^2}{s}\right)F_0(A,B)+\frac{1}{2s}\left(M_B^2\log M_B^2-M_A^2\log M_A^2-M_B^2+M_A^2\right) \tag{243}$$

$$\begin{aligned}F_2(A,B) = & \; \frac{2}{3}\left(1+\frac{M_A^2-M_B^2}{s}\right)F_1(A,B)-\frac{M_A^2}{3s}F_0(A,B)\\ & +\frac{1}{3s}\left(M_B^2\log M_B^2+\frac{1}{2}(M_A^2-M_B^2)\right)-\frac{1}{18}\end{aligned} \tag{244}$$

Reduction into F_0, $A=B$

$$F_1(A,A) = \frac{1}{2}F_0(A,A) \tag{245}$$

$$F_2(A,A) = \frac{1}{3}\left(1 - \frac{M_A^2}{s}\right) F_0(A,A) + \frac{M_A^2}{3s}\log M_A^2 - \frac{1}{18} \tag{246}$$

G functions (Derivative of F)

$$G_n(A,B) = \frac{d}{ds}F_n(A,B) = \int_0^1 \frac{-x^n \cdot x(1-x)}{D_2}dx \tag{247}$$

F and G at special energy

$$\begin{aligned} F_n(A,B,C) &= F_n(A,B)|_{s=M_C^2} \\ F_n(A,B,0) &= F_n(A,B)|_{s=0} \\ G_n(A,B,C) &= G_n(A,B)|_{s=M_C^2} \\ G_n(A,B,0) &= G_n(A,B)|_{s=0} \end{aligned} \tag{248}$$

F_0 for $s=0$

$$F_0(A,B,0) = \begin{cases} \log M_A^2 & (A=B) \\ \dfrac{M_B^2\log M_B^2 - M_A^2\log M_A^2}{M_B^2 - M_A^2} - 1 & (A \neq B) \end{cases} \tag{249}$$

9.2 $Z - Z$

(a)	$(A,B) = (W,W),(W,\chi),(\chi,W),(\chi,\chi),(H,\chi),(H,Z),(c^+,c^+),(c^-,c^-),(f,f)$
(b)	$A = W, H, \chi_3, \chi, c^+, c^-$

The second lines of Π_T^{ZZ} and Π_L^{ZZ} show fermionic contribution.

$$\begin{aligned} \Pi_T^{ZZ}(s) = \; & \frac{\alpha}{4\pi c_W^2 s_W^2}\Big[C^{ZZ,T}C_{\text{UV}} + d_{WW}^{ZZ,T}F_0(W,W) + d_{HZ}^{ZZ,T}F_0(H,Z) + d_0^{ZZ,T} \\ & -\frac{1}{2}\sum_f\Big[\Big\{(2I_3 - 4Q_f s_w^2)^2 + 1\Big\}\left(\frac{1}{6}C_{\text{UV}} - F_{12}(f,f)\right)s \\ & -m_f^2\left(C_{\text{UV}} - F_0(f,f)\right)\Big]\Big] \end{aligned} \tag{250}$$

$$C^{ZZ,T} = \left(3c_W^4 + \frac{1}{3}c_W^2 - \frac{1}{6}\right)s + 4c_W^2M_W^2 - 2M_W^2 - M_Z^2 + 4c_W^2\tilde{\beta}(c_W^2 s - M_W^2) \tag{251}$$

$$d_{WW}^{ZZ,T} = \left(-3c_W^4 - \frac{1}{3}c_W^2 + \frac{1}{12}\right)s - 4c_W^4M_W^2 - \frac{8}{3}c_W^2M_W^2 + \frac{5}{3}M_W^2 + 4c_W^2\tilde{\beta}(-c_W^2 s + M_W^2) \tag{252}$$

$$d_{HZ}^{ZZ,T} = \frac{1}{12s}(M_H^2 - M_Z^2)^2 + \frac{1}{12}s - \frac{1}{6}M_H^2 + \frac{5}{6}M_Z^2 \tag{253}$$

$$d_0^{ZZ,T} = M_W^2 \log M_W^2 \left(4c_W^4 - \frac{4}{3}c_W^2 + \frac{1}{3}\right) + M_H^2 \log M_H^2 \left(-\frac{(M_H^2 - M_Z^2)}{12s} + \frac{1}{6}\right)$$
$$+M_Z^2 \log M_Z^2 \left(\frac{(M_H^2 - M_Z^2)}{12s} + \frac{1}{6}\right) + \frac{(M_H^2 - M_Z^2)^2}{12s} + \left(\frac{2}{9}c_W^2 - \frac{1}{9}\right) s \tag{254}$$

$$\Pi_L^{ZZ}(s) = \frac{\alpha}{4\pi c_W^2 s_W^2} \Big[C^{ZZ,L} C_{\mathsf{UV}} + d_{WW}^{ZZ,L} F_0(W,W) + d_{HZ}^{ZZ,L} F_0(H,Z) + d_0^{ZZ,L}$$
$$+\frac{1}{2}\sum_f \left\{m_f^2 \left(C_{\mathsf{UV}} - F_0(f,f)\right)\right\}\Big] \tag{255}$$

$$C^{ZZ,L} = 4c_W^2 M_W^2 - 2M_W^2 - M_Z^2 - 4\tilde{\beta} c_W^2 M_W^2 + \frac{1}{4}\tilde{\varepsilon}^2 s \tag{256}$$

$$d_{WW}^{ZZ,L} = -4c_W^2 M_W^2 + 2M_W^2 + 4\tilde{\beta} c_W^2 M_W^2 \tag{257}$$

$$d_{HZ}^{ZZ,L} = -\frac{(M_H^2 - M_Z^2)^2}{4s} + M_Z^2 + \frac{\tilde{\varepsilon}}{2}(-M_H^2 + M_Z^2) - \frac{\tilde{\varepsilon}^2}{4}s \tag{258}$$

$$d_0^{ZZ,L} = (\log M_H^2 - 1) M_H^2 \left(\frac{(M_H^2 - M_Z^2)}{4s} + \frac{\tilde{\varepsilon}}{2}\right)$$
$$+(\log M_Z^2 - 1) M_Z^2 \left(-\frac{(M_H^2 - M_Z^2)}{4s} - \frac{\tilde{\varepsilon}}{2}\right) \tag{259}$$

9.3 $Z - A$

(a)	$(A,B) = (W,W), (W,\chi), (\chi,W), (\chi,\chi), (c^+,c^+), (c^-,c^-), (f,f)$
(b)	$A = W, \chi, c^+, c^-$

The second line of Π_T^{ZA} shows fermionic contribution. No fermionic contribution appears in Π_L^{ZA}.

$$\Pi_T^{ZA}(s) = \frac{\alpha c_W}{4\pi s_W} \Big[C^{ZA,T} C_{\mathsf{UV}} + d_{WW}^{ZA,T} F_0(W,W) + d_0^{ZA,T}$$
$$-\frac{2}{c_W^2}\sum_f Q_f \left(2I_3 - 4Q_f s_w^2\right)\left(\frac{1}{6}C_{\mathsf{UV}} - F_{12}(f,f)\right) s\Big] \tag{260}$$

$$C^{ZA,T} = \left(\frac{1}{6c_W^2} + 3\right) s + 2M_Z^2 + 2\tilde{\alpha}(s - M_Z^2) + 2\tilde{\beta} s \tag{261}$$

$$d_{WW}^{ZA,T} = \left(-\frac{1}{6c_W^2} - 3\right) s - \frac{4}{3}M_Z^2 - 4M_W^2 - 2\tilde{\alpha}(s - M_Z^2) - 2\tilde{\beta} s \tag{262}$$

$$d_0^{ZA,T} = \log M_W^2 \left(-\frac{2}{3}M_Z^2 + 4M_W^2\right) + \frac{1}{9c_W^2}s \tag{263}$$

$$\Pi_L^{ZA}(s) = \frac{\alpha c_W}{4\pi s_W} 2(1-\tilde{\alpha})M_Z^2\left(C_{\mathsf{UV}} - F_0(W,W)\right) \tag{264}$$

9.4 $A - A$

(a)	$(A,B) = (W,W),(W,\chi),(\chi,W),(\chi,\chi),(c^+,c^+),(c^-,c^-),(f,f)$
(b)	$A = W,\chi,c^+,c^-$

The second line of Π_T^{AA} shows fermionic contribution. Longitudinal part Π_L^{AA} is exactly 0.

$$\begin{aligned}\Pi_T^{AA}(s) = \quad & \frac{\alpha}{4\pi}\left[C^{AA,T}C_{\mathsf{UV}} + d_{WW}^{AA,T}F_0(W,W) + d_0^{AA,T}\right.\\ & \left. -8\sum_f Q_f^2\left(\frac{1}{6}C_{\mathsf{UV}} - F_{12}(f,f)\right)s\right]\end{aligned} \tag{265}$$

$$C^{AA,T} = 3s + 4\tilde{\alpha}s \tag{266}$$

$$d_{WW}^{AA,T} = -3s - 4M_W^2 - 4\tilde{\alpha}s \tag{267}$$

$$d_0^{AA,T} = 4M_W^2\log M_W^2 \tag{268}$$

$$\Pi_L^{AA}(s) = 0 \tag{269}$$

9.5 $W - W$

(a)	$(A,B) = (Z,W),(Z,\chi),(A,W),(A,\chi),(H,\chi),(H,W),(\chi_3,\chi),$ $(c^Z,c^+),(c^Z,c^-),(c^A,c^+),(c^A,c^-),(f,f')$
(b)	$A = A,Z,W,H,\chi_3,\chi,c^+,c^-$

The last lines of Π_T^{WW} and Π_L^{WW} shows fermionic contribution.

$$\begin{aligned}&\Pi_T^{WW}(s) = \\ &\quad \frac{\alpha}{4\pi s_W^2}\left[C^{WW,T}C_{\mathsf{UV}} + d_{ZW}^{WW,T}F_0(Z,W) + d_{HW}^{WW,T}F_0(H,W) + d_{AW}^{WW,T}F_0(A,W) + d_0^{WW,T}\right.\\ &\quad -\frac{1}{2}\sum_{doublet}\left\{4\left(\frac{1}{6}C_{\mathsf{UV}} - F_{12}(f,f')\right)s\right.\\ &\quad \left.\left. -(m_f^2 + m_f'^2)C_{\mathsf{UV}} + 2m_f^2F_1(f',f) + 2m_f'^2F_1(f,f')\right\}\right]\end{aligned} \tag{270}$$

$$C^{WW,T} = \frac{19}{6}s + 2M_W^2 - M_Z^2 + 2\tilde{\alpha}s_W^2(M_W^2 - s) + 2\tilde{\beta}c_W^2(M_W^2 - s) \tag{271}$$

$$\begin{aligned} d_{ZW}^{WW,T} = \; & \frac{(1+8c_W^2)(M_Z^2 - M_W^2)^2}{12s} + \left(\frac{1}{12} - \frac{10}{3}c_W^2\right)s - \frac{4}{3}c_W^2 M_W^2 - \frac{9}{2}M_W^2 + \frac{5}{6}M_Z^2 \\ & + 2\tilde{\beta}c_W^2(s - M_W^2) \end{aligned} \tag{272}$$

$$d_{HW}^{WW,T} = \frac{(M_H^2 - M_W^2)^2}{12s} + \frac{1}{12}s - \frac{1}{6}M_H^2 + \frac{5}{6}M_W^2 \tag{273}$$

$$d_{AW}^{WW,T} = \frac{2s_W^2(M_W^2)^2}{3s} - \frac{10}{3}s_W^2 s - \frac{4}{3}s_W^2 M_W^2 + 2\tilde{\alpha}s_W^2(s - M_W^2) \tag{274}$$

$$\begin{aligned} d_0^{WW,T} = \; & \log M_W^2 \left(\frac{5}{3}M_W^2 + \frac{M_W^2(M_H^2 + M_Z^2 - 2M_W^2)}{12s}\right) \\ & + \log M_H^2 \left(-\frac{(M_H^2 - M_W^2)M_H^2}{12s} + \frac{1}{6}M_H^2\right) \\ & + \log M_Z^2 \left(-\frac{(M_Z^2 - M_W^2)(M_Z^2 + 8M_W^2)}{12s} + \frac{4}{3}M_W^2 + \frac{1}{6}M_Z^2\right) \\ & + \frac{(M_Z^2 - M_W^2)(M_Z^2 + 7M_W^2)}{12s} + \frac{(M_H^2 - M_W^2)^2}{12s} + \frac{1}{9}s \end{aligned} \tag{275}$$

$$\begin{aligned} & \Pi_L^{WW}(s) = \\ & \frac{\alpha}{4\pi s_W^2}\Bigg[C^{WW,L}C_{\mathsf{UV}} + d_{ZW}^{WW,L}F_0(Z,W) + d_{HW}^{WW,L}F_0(H,W) + d_{AW}^{WW,L}F_0(A,W) + d_0^{WW,L} \\ & -\frac{1}{2}\sum_{doublet}\left\{-(m_f^2 + m_f'^2)C_{\mathsf{UV}} + 2m_f^2F_1(f',f) + 2m_f'^2F_1(f,f')\right\}\Bigg] \end{aligned} \tag{276}$$

$$C^{WW,L} = 2M_W^2 - M_Z^2 + 2\tilde{\alpha}s_W^2M_W^2 + 2\tilde{\beta}c_W^2M_W^2 + 5\tilde{\alpha}^2s_W^2s + 5\tilde{\beta}^2c_W^2s + \frac{1}{4}\tilde{\delta}^2s + \frac{1}{4}\tilde{\kappa}^2s \tag{277}$$

$$\begin{aligned} d_{ZW}^{WW,L} = \; & -\frac{(1+8c_W^2)(M_Z^2 - M_W^2)^2}{4s} - 3M_W^2 + M_Z^2 \\ & + \tilde{\beta}(4c_W^2 - 6)M_W^2 + \frac{1}{2}\tilde{\kappa}(M_W^2 - M_Z^2) + \tilde{\beta}^2(M_W^2 - c_W^2M_W^2 - 5c_W^2s) - \frac{1}{4}\tilde{\kappa}^2s \end{aligned} \tag{278}$$

$$d_{HW}^{WW,L} = -\frac{(M_H^2 - M_W^2)^2}{4s} + M_W^2 + \frac{1}{2}\tilde{\delta}(M_W^2 - M_H^2) - \frac{1}{4}\tilde{\delta}^2s \tag{279}$$

$$d_{AW}^{WW,L} = -\frac{2s_W^2(M_W^2)^2}{s} + 4\tilde{\alpha}s_W^2 M_W^2 - \tilde{\alpha}^2 s_W^2(M_W^2 + 5s) \tag{280}$$

$$\begin{aligned} d_0^{WW,L} = \; & \log M_W^2 \Big(\frac{(2M_W^2 - M_H^2 - M_Z^2)M_W^2}{4s} \\ & + \left(-6\tilde{\alpha}s_W^2 - 6\tilde{\beta}c_W^2 + \tilde{\alpha}^2 s_W^2 + \tilde{\beta}^2 c_W^2 - \frac{1}{2}\tilde{\delta} - \frac{1}{2}\tilde{\kappa} \right) M_W^2 \Big) \\ & + \log M_H^2 \left(\frac{(M_H^2 - M_W^2)M_H^2}{4s} + \frac{1}{2}\tilde{\delta}M_H^2 \right) \\ & + \log M_Z^2 \left(\frac{(M_Z^2 - M_W^2)(M_Z^2 + 8M_W^2)}{4s} + 6\tilde{\beta}M_W^2 - \tilde{\beta}^2 M_W^2 + \frac{1}{2}\tilde{\kappa}M_Z^2 \right) \\ & - \frac{(M_H^2 - M_W^2)^2 + (M_Z^2 - M_W^2)(M_Z^2 + 7M_W^2)}{4s} + 6\tilde{\alpha}s_W^2 M_W^2 - 6\tilde{\beta}s_W^2 M_W^2 \\ & - \tilde{\alpha}^2 s_W^2(2s + M_W^2) + \tilde{\beta}^2(s_W^2 M_W^2 - 2c_W^2 s) + \frac{1}{2}\tilde{\delta}(M_W^2 - M_H^2) + \frac{1}{2}\tilde{\kappa}(M_W^2 - M_Z^2) \end{aligned} \tag{281}$$

9.6 $H - H$

(a)	$(A, B) = (W, W), (W, \chi), (\chi, W), (\chi, \chi), (Z, \chi_3), (\chi_3, \chi_3), (Z, Z), (H, H),$ $(c^+, c^+), (c^-, c^-), (c^Z, c^Z), (f, f)$
(b)	$A = W, Z, H, \chi, \chi_3, c^+, c^-, c^Z$

The second line of Π^H shows fermionic contribution. Since the tadpole contribution appears with Π^H, we present the formula for the sum.

$$\begin{aligned} \Pi^H(s) + \frac{3\delta T}{v} = \; & \frac{\alpha}{4\pi s_W^2} \Big[C^H C_{\mathsf{UV}} + d_{WW}^H F_0(W, W) + d_{ZZ}^H F_0(Z, Z) + d_{HH}^H F_0(H, H) + d_0^H \\ & + \sum_f \frac{m_f^2}{M_W^2} \left\{ \frac{s}{2} C_{\mathsf{UV}} + 2m_f^2(1 - \log m_f^2) - \frac{1}{2}(s - 4m_f^2)F_0(f, f) \right\} \Big] \end{aligned} \tag{282}$$

$$C^H = -\left(\frac{1}{2c_W^2} + 1 \right) s - \frac{1}{4c_W^2} M_H^2 + \frac{3}{4}\frac{(M_H^2)^2}{M_W^2} - \frac{1}{2}M_H^2 + \tilde{\delta}(M_H^2 - s) + \frac{1}{2c_W^2}\tilde{\varepsilon}(M_H^2 - s) \tag{283}$$

$$d_{WW}^H = s - 3M_W^2 - \frac{1}{4}\frac{(M_H^2)^2}{M_W^2} + \tilde{\delta}(s - M_H^2) \tag{284}$$

$$d_{ZZ}^H = \frac{1}{2}c_W^2 s - \frac{3}{2c_W^2}M_Z^2 - \frac{1}{8}\frac{(M_H^2)^2}{M_W^2} + \frac{1}{2c_W^2}\tilde{\varepsilon}(s - M_H^2) \tag{285}$$

$$d_{HH}^{H} = -\frac{9}{8}\frac{(M_H^2)^2}{M_W^2} \tag{286}$$

$$\begin{aligned} d_0^H = & \log M_W^2\left(\frac{1}{2}M_H^2 + 3M_W^2\right) + \log M_H^2\left(\frac{3(M_H^2)^2}{4M_W^2}\right) \\ & + \log M_Z^2\left(\frac{M_H^2}{4c_W^2} + \frac{3M_Z^2}{2c_W^2}\right) - \frac{M_H^2}{4c_W^2} - \frac{3M_Z^2}{2c_W^2} - \frac{3(M_H^2)^2}{4M_W^2} - \frac{1}{2}M_H^2 - 3M_W^2 \end{aligned} \tag{287}$$

9.7 $\chi - \chi$

(a)	$(A,B) = (H,W), (H,\chi), (\chi_3,W), (Z,\chi), (Z,W), (A,\chi), (A,W), (c^Z,c^+), (c^Z,c^-), (f,f')$
(b)	$A = A, Z, W, H, \chi_3, \chi, c^+, c^-$

The second line of Π^χ shows fermionic contribution. Since the tadpole contribution appears with Π^χ, we present the formula for the sum. It is observed that C^χ, coefficient for the divergent part, is proportional to s in the linear gauge.

$$\begin{aligned} \Pi^\chi(s) + \frac{\delta T}{v} = & \frac{\alpha}{16\pi s_W^2}\Big[C^\chi C_{\mathsf{UV}} + d_{ZW}^\chi F_0(Z,W) + d_{HW}^\chi F_0(H,W) + d_{AW}^\chi F_0(A,W) + d_0^\chi \\ & +2s\sum_f \frac{m_f^2}{M_W^2}\left(C_{\mathsf{UV}} - 2F_1(f',f)\right)\Big] \end{aligned} \tag{288}$$

$$\begin{aligned} C^\chi = & -\left(\frac{2}{c_W^2} + 4\right)s - 32\tilde{\alpha}s_W^2M_W^2 + 32\tilde{\beta}s_W^2M_W^2 + 16\tilde{\alpha}^2s_W^2M_W^2 + 16\tilde{\beta}^2c_W^2M_W^2 \\ & +2\tilde{\delta}(s + M_H^2) + 3\tilde{\delta}^2M_W^2 + 2\tilde{\kappa}s - \tilde{\kappa}^2M_W^2 \end{aligned} \tag{289}$$

$$\begin{aligned} d_{ZW}^\chi = & \left(2 - 8s_W^2 + \frac{2}{c_W^2}\right)s - 16c_W^2M_W^2 - \frac{6}{c_W^2}M_W^2 - \frac{1}{c_W^2}M_Z^2 - 8s_W^2M_W^2 + 23M_W^2 \\ & -32\tilde{\beta}s_W^2M_W^2 - 16\tilde{\beta}^2c_W^2M_W^2 - 2\tilde{\kappa}s + \tilde{\kappa}^2M_W^2 \end{aligned} \tag{290}$$

$$d_{HW}^\chi = 2s - M_W^2 + 2M_H^2 - \frac{(M_H^2)^2}{M_W^2} - 2\tilde{\delta}(s + M_H^2) - 3\tilde{\delta}^2M_W^2 \tag{291}$$

$$d_{AW}^\chi = 8s_W^2(s - M_W^2) + 32\tilde{\alpha}s_W^2M_W^2 - 16\tilde{\alpha}^2s_W^2M_W^2 \tag{292}$$

$$\begin{aligned} d_0^\chi = & \log M_W^2\left(-M_H^2 + 2M_W^2 - M_Z^2\right) + \log M_H^2\left(\frac{(M_H^2)^2}{M_W^2} - M_H^2\right) \\ & + \log M_Z^2\left(\frac{M_Z^2}{c_W^2} + 8s_W^2M_Z^2 - M_Z^2\right) - \frac{(M_H^2)^2}{M_W^2} + 2M_H^2 + 6M_W^2 - \frac{M_Z^2}{c_W^2} - 6M_Z^2 \\ & +16\tilde{\alpha}s_W^2M_W^2 - 16\tilde{\beta}s_W^2M_W^2 - 8\tilde{\alpha}^2s_W^2M_W^2 - 8\tilde{\beta}^2c_W^2M_W^2 \end{aligned} \tag{293}$$

9.8 $\chi_3 - \chi_3$

(a)	$(A,B) = (W,\chi),(\chi,W),(H,Z),(H,\chi_3),(c^+,c^+),(c^-,c^-),(f,f)$
(b)	$A = W, Z, \chi_3, \chi, H, c^+, c^-, c^Z$

The second line of $\Pi^{\chi 3}$ shows fermionic contribution. Since the tadpole contribution appears with $\Pi^{\chi 3}$, we present the formula for the sum. It is observed that $C^{\chi 3}$, coefficient for the divergent part, is proportional to s in the linear gauge.

$$\begin{aligned}\Pi^{\chi 3}(s) + \frac{\delta T}{v} = & \frac{\alpha}{16\pi s_W^2}\Big[C^{\chi 3}C_{\mathsf{UV}} + d^{\chi 3}_{WW}F_0(W,W) + d^{\chi 3}_{HZ}F_0(H,Z) + d_0^{\chi 3} \\ & +2s\sum_f \frac{m_f^2}{M_W^2}\left(C_{\mathsf{UV}} - F_0(f,f)\right)\Big]\end{aligned} \tag{294}$$

$$C^{\chi 3} = -\left(\frac{2}{c_W^2}+4\right)s + \tilde{\varepsilon}\frac{2}{c_W^2}(M_H^2+s) + \tilde{\varepsilon}^2\frac{3}{c_W^2}M_Z^2 - 4\tilde{\kappa}s \tag{295}$$

$$d^{\chi 3}_{WW} = 4s + 4\tilde{\kappa}s \tag{296}$$

$$d^{\chi 3}_{HZ} = \frac{1}{c_W^2}\left(2M_H^2 - M_Z^2 + 2s - \frac{(M_H^2)^2}{M_Z^2} - 2\tilde{\varepsilon}(M_H^2+s) - 3\tilde{\varepsilon}^2 M_Z^2\right) \tag{297}$$

$$d_0^{\chi 3} = \frac{1}{c_W^2}\left[\log M_H^2\left(\frac{(M_H^2)^2}{M_Z^2} - M_H^2\right) + \log M_Z^2\left(-M_H^2+M_Z^2\right) - \frac{(M_H^2)^2}{M_Z^2} + 2M_H^2 - M_Z^2\right] \tag{298}$$

9.9 $Z - \chi_3$

(a)	$(A,B) = (W,\chi),(\chi,W),(H,\chi_3),(H,Z),(c^+,c^+),(c^-,c^-),(f,f)$
(b)	None

The second line of $\Pi^{Z\chi 3}$ shows fermionic contribution.

$$\begin{aligned}\Pi^{Z\chi 3}(s) = & \frac{\alpha M_Z}{8\pi s_W^2 c_W^2}\Big[C^{Z\chi 3}C_{\mathsf{UV}} + d^{Z\chi 3}_{WW}F_0(W,W) + d^{\chi 3}_{HZ}F_0(H,Z) + d_0^{Z\chi 3} \\ & +\sum_f \frac{m_f^2}{M_Z^2}(C_{\mathsf{UV}} - F_0(f,f))\Big]\end{aligned} \tag{299}$$

$$C^{Z\chi 3} = \left(-\frac{3}{2} - 3c_W^2 + 4c_W^4\right) - 4\tilde{\beta}c_W^4 + \frac{1}{2}\tilde{\varepsilon}\left(1+\frac{M_H^2}{M_Z^2}\right) + \tilde{\varepsilon}^2 - \tilde{\kappa}c_W^2 \tag{300}$$

$$d^{Z\chi 3}_{WW} = 3c_W^2 - 4c_W^4 + 4\tilde{\beta}c_W^4 + \tilde{\kappa}c_W^2 \tag{301}$$

$$d^{Z\chi 3}_{HZ} = -\frac{(M_H^2-M_Z^2)^2}{2sM_Z^2} + \frac{3}{2} + \tilde{\varepsilon}\left(-\frac{(M_H^2-M_Z^2)}{2s} - \frac{1}{2} - \frac{M_H^2}{2M_Z^2}\right) - \tilde{\varepsilon}^2 \tag{302}$$

$$
\begin{aligned}
d_0^{Z\chi 3} = \quad & (\log M_H^2 - 1)\frac{M_H^2}{2s}\left(\frac{(M_H^2 - M_Z^2)}{M_Z^2} + \tilde{\varepsilon}\right) \\
& +(\log M_Z^2 - 1)\frac{M_Z^2}{2s}\left(-\frac{(M_H^2 - M_Z^2)}{M_Z^2} - \tilde{\varepsilon}\right)
\end{aligned} \tag{303}
$$

9.10 $A - \chi_3$

(a)	$(A, B) = (W, \chi), (\chi, W), (c^+, c^+), (c^-, c^-), (f, f)$
(b)	None

There is no fermionic contribution.

$$
\Pi^{A\chi 3}(s) = \frac{\alpha M_W}{2\pi s_W}(1 - \tilde{\alpha})\left(C_{\mathsf{UV}} - F_0(W, W)\right) \tag{304}
$$

9.11 $W - \chi$

(a)	$(A, B) = (H, W), (H, \chi), (Z, \chi), (Z, W), (A, \chi), (A, W),$ $(c^A, c), (c^Z, c^+), (c^Z, c^-), (f, f')$
(b)	None

The second line of $\Pi^{W\chi}$ shows fermionic contribution.

$$
\begin{aligned}
\Pi^{W\chi}(s) = \quad & \frac{\alpha M_W}{16\pi s_W^2}\Bigg[C^{W\chi}C_{\mathsf{UV}} + d_{ZW}^{W\chi}F_0(Z, W) + d_{HW}^{\chi 3}F_0(H, W) + d_{AW}^{W\chi}F_0(A, W) + d_0^{W\chi} \\
& +4\sum_{doublet}\frac{1}{M_W^2}\left\{\frac{m_f^2 + m_f'^2}{2}C_{\mathsf{UV}} - \left(m_f'^2 F_0(f', f) + (m_f^2 - m_f'^2)F_1(f', f)\right)\right\}\Bigg]
\end{aligned} \tag{305}
$$

$$
\begin{aligned}
C^{W\chi} = \quad & -\frac{3}{c_W^2} + 2 \\
& -12\tilde{\alpha}s_W^2 + \tilde{\beta}(16 - 12c_W^2) + 18\tilde{\alpha}^2 s_W^2 + 18\tilde{\beta}^2 c_W^2 \\
& +\tilde{\delta}\left(1 + \frac{M_H^2}{M_W^2}\right) + \tilde{\kappa} + 2\tilde{\delta}^2
\end{aligned} \tag{306}
$$

$$
\begin{aligned}
d_{ZW}^{W\chi} = \quad & -\frac{(1 + 8c_W^2)s_W^2(M_Z^2 - M_W^2)}{c_W^2 s} - 4c_W^2 + 3\frac{s_W^2}{c_W^2} - 8s_W^2 + 2 \\
& +\tilde{\beta}\left(-12c_W^2\frac{M_Z^2 - M_W^2}{s} - 16 + 12c_W^2\right) + \tilde{\beta}^2\left(2c_W^2\frac{M_Z^2 - M_W^2}{s} - 18c_W^2\right) \\
& +\tilde{\kappa}\left(\frac{M_Z^2 - M_W^2}{c_W^2 s} - 1\right)
\end{aligned} \tag{307}
$$

$$d_{HW}^{W\chi} = -\frac{(M_H^2 - M_W^2)^2}{sM_W^2} + 3 + \tilde{\delta}\left(-\frac{M_H^2 - M_W^2}{s} - \frac{M_H^2}{M_W^2} - 1\right) - 2\tilde{\delta}^2 \quad (308)$$

$$d_{AW}^{W\chi} = -8\frac{s_W^2 M_W^2}{s} + 4s_W^2 + 12\tilde{\alpha}s_W^2\left(1 + \frac{M_W^2}{s}\right) + \tilde{\alpha}^2 s_W^2\left(-18 - \frac{2M_W^2}{s}\right) \quad (309)$$

$$\begin{aligned}
d_0^{W\chi} = \; & \log M_W^2\left(-\frac{M_Z^2 + M_H^2 - 2M_W^2}{s} - 12(\tilde{\alpha}s_W^2 + \tilde{\beta}c_W^2)\frac{M_W^2}{s} + 2(\tilde{\alpha}^2 s_W^2 + \tilde{\beta}^2 c_W^2)\frac{M_W^2}{s}\right) \\
& + \log M_H^2\left(\frac{M_H^2(M_H^2 - M_W^2)}{sM_W^2} + \tilde{\delta}\frac{M_H^2}{s}\right) \\
& + \log M_Z^2\left(\frac{8s_W^2 M_W^2 + s_W^2 M_Z^2}{c_W^2 s} + 12\tilde{\beta}\frac{M_W^2}{s} - 2\tilde{\beta}^2\frac{M_W^2}{s} + \tilde{\kappa}\frac{M_Z^2}{s}\right) \\
& -7\frac{s_W^2 M_Z^2}{s} - \frac{s_W^2 M_Z^2}{c_W^2 s} - \frac{(M_H^2 - M_W^2)^2}{sM_W^2} - \tilde{\delta}\frac{M_H^2 - M_W^2}{s} - \tilde{\kappa}\frac{M_Z^2 - M_W^2}{s} \\
& +4\tilde{\alpha}s_W^2\left(3\frac{M_W^2}{s} + 2\right) - 4\tilde{\beta}s_W^2\left(3\frac{M_W^2}{s} + 2\right) - 2\tilde{\alpha}^2 s_W^2\left(\frac{M_W^2}{s} + 4\right) + 2\tilde{\beta}^2\left(\frac{s_W^2 M_W^2}{s} - 4c_W^2\right)
\end{aligned} \quad (310)$$

9.12 $H - Z, A, \chi_3$

$H - Z$	
(a)	$(A, B) = (W, W), (W, \chi), (\chi, W), (\chi, \chi), (c^+, c^+), (c^-, c^-), (f, f)$
(b)	None
$H - A$	
(a)	$(A, B) = (W, W), (W, \chi), (\chi, W), (\chi, \chi), (c^+, c^+), (c^-, c^-), (f, f)$
(b)	None
$H - \chi_3$	
(a)	$(A, B) = (W, \chi), (\chi, W), (c^+, c^+), (c^-, c^-), (f, f)$
(b)	$A = c^+, c^-$

For these two-point functions, there are some diagrams as in the above table. However, the explicit calculation shows that the total result is 0 for these 3 two-point functions. This holds for both in the linear gauge case and in the non-linear one. Contribution of fermion loop is 0 by itself.

The vanishing of these functions can be understood as follows. The higgs field H can acquire the vacuum expectation value. Since the vacuum is scalar, these functions should be 0 in order to avoid the case where a vector or pseudoscalar field has non-zero vacuum expectation value.

9.13 $f - f$

(a)	$(A, B) = (f, A), (f, Z), (f', W), (f, H), (f, \chi_3), (f', \chi)$
(b)	None

By the CP-invariance condition $K_5^f = 0$ holds.

$$K_j^f(s) = \frac{\alpha}{4\pi}\left[Q_f^2 K_j^A + \frac{1}{c_w^2} Q_f^2 s_w^2 K_j^{Z(1)} \mp \frac{1}{2c_w^2} Q_f K_j^{Z(2)} + \frac{1}{8s_w^2 c_w^2} K_j^{Z(3)} \right.$$
$$\left. + \frac{1}{4s_w^2} K_j^W + \frac{1}{4s_w^2 c_w^2}\frac{m_f^2}{M_Z^2} K_j^S\right] \qquad (j = 1, \gamma, 5\gamma) \tag{311}$$

$$\begin{aligned} K_1^A &= m_f[-4C_{\mathsf{UV}} + 2 + 4F_0(f, A)] \\ K_\gamma^A &= C_{\mathsf{UV}} - 1 - 2F_1(f, A) \\ K_{5\gamma}^A &= 0 \end{aligned} \tag{312}$$

$$\begin{aligned} &K_1^{Z(1)} = m_f[-4C_{\mathsf{UV}} + 2 + 4F_0(f, Z)], \quad K_1^{Z(2)} = K_1^{Z(1)}, \quad K_1^{Z(3)} = 0 \\ &K_\gamma^{Z(1)} = C_{\mathsf{UV}} - 1 - 2F_1(f, Z), \quad K_\gamma^{Z(2)} = K_\gamma^{Z(1)}, \quad K_\gamma^{Z(3)} = K_\gamma^{Z(1)} \\ &K_{5\gamma}^{Z(1)} = 0, \quad K_{5\gamma}^{Z(2)} = -K_\gamma^{Z(1)}, \quad K_{5\gamma}^{Z(3)} = -K_\gamma^{Z(1)} \end{aligned} \tag{313}$$

$$\begin{aligned} K_1^W &= 0 \\ K_\gamma^W &= C_{\mathsf{UV}} - 1 - 2F_1(f', W) \\ K_{5\gamma}^W &= -K_\gamma^W \end{aligned} \tag{314}$$

$$\begin{aligned} K_1^S &= m_f\left[-F_0(f, H) + F_0(f, Z) - 2\frac{m_f'^2}{m_f^2}(C_{\mathsf{UV}} - F_0(f', W))\right] \\ K_\gamma^S &= C_{\mathsf{UV}} - F_1(f, H) - F_1(f, Z) + \frac{1}{2}\left(1 + \frac{m_f'^2}{m_f^2}\right)(C_{\mathsf{UV}} - 2F_1(f', W)) \\ K_{5\gamma}^S &= +\frac{1}{2}\left(1 - \frac{m_f'^2}{m_f^2}\right)(C_{\mathsf{UV}} - 2F_1(f', W)) \end{aligned} \tag{315}$$

10 Renormalization constants

10.1 Vector part

$\underline{\delta Z^{1/2}_{AA}}$

$$\delta Z^{1/2}_{AA} = \frac{\alpha}{4\pi}\left[\left(\frac{3}{2}+2\tilde{\alpha}\right)C_{\mathsf{UV}} - \left(\frac{3}{2}+2\tilde{\alpha}\right)\log M_W^2 + \frac{1}{3} - \frac{2}{3}\sum_f Q_f^2\left(C_{\mathsf{UV}} - \log m_f^2\right)\right] \tag{316}$$

$\underline{\delta Z^{1/2}_{ZA}}$

$$\delta Z^{1/2}_{ZA} = \frac{\alpha c_W}{4\pi s_W}\left\{(-2+2\tilde{\alpha})\,C_{\mathsf{UV}} + (2-2\tilde{\alpha})\log M_W^2\right\} \tag{317}$$

(No fermion term appears in $\delta Z^{1/2}_{ZA}$.)

$\underline{\delta Z^{1/2}_{AZ}}$

$$\delta Z^{1/2}_{AZ} = \frac{\alpha c_W}{4\pi s_W}\left[\left(\frac{1}{6c_W^2}+5+2\tilde{\beta}\right) - \sum_f \frac{1}{3}Q_f\left(2I_3 - 4Q_f s_w^2\right)\right]C_{\mathsf{UV}} + \delta Z^{1/2}_{AZ,fin} \tag{318}$$

$$\begin{aligned}\delta Z^{1/2}_{AZ,fin} = {} & \frac{\alpha c_W}{4\pi s_W}\Big[F_0(W,W,Z)\left(-4c_W^2 - \frac{1}{6c_W^2} - \frac{13}{3} - 2\tilde{\beta}\right) + \log M_W^2\left(4c_W^2 - \frac{2}{3}\right) \\ & + \frac{1}{9c_W^2} + 2\sum_f Q_f\left(2I_3 - 4Q_f s_w^2\right)F_{12}(f,f,Z)\Big]\end{aligned} \tag{319}$$

$\underline{\delta Z^{1/2}_{ZZ}}$

$$\begin{aligned}\delta Z^{1/2}_{ZZ} = {} & \frac{\alpha}{4\pi s_W^2 c_W^2}\left[\frac{3}{2}c_W^4 + \frac{1}{6}c_W^2 - \frac{1}{12} + 2c_W^4\tilde{\beta} - \frac{1}{24}\sum_f\left\{(2I_3 - 4Q_f s_w^2)^2 + 1\right\}\right]C_{\mathsf{UV}} \\ & + \delta Z^{1/2}_{ZZ,fin,b} + \delta Z^{1/2}_{ZZ,fin,f}\end{aligned} \tag{320}$$

$$\begin{aligned}
\delta Z^{1/2}_{ZZ,fin,b} = \quad & \frac{\alpha}{4\pi s_W^2 c_W^2}\Big[F_0(W,W,Z)\left(-\frac{3}{2}c_W^4 - \frac{1}{6}c_W^2 + \frac{1}{24} - 2c_W^4\tilde{\beta}\right) \\
& +G_0(W,W,Z)\left(-2c_W^4 - \frac{17}{6}c_W^2 + \frac{2}{3} + \frac{1}{24c_W^2}\right)M_W^2 \\
& +F_0(H,Z,Z)\left(-\frac{1}{24}\frac{M_H^4}{M_Z^4} + \frac{1}{12}\frac{M_H^2}{M_Z^2}\right) \\
& +G_0(H,Z,Z)\left(+\frac{1}{24}\frac{M_H^4}{M_Z^4} - \frac{1}{6}\frac{M_H^2}{M_Z^2} + \frac{1}{2}\right)M_Z^2 \\
& +\log M_H^2\left(+\frac{1}{24}\frac{M_H^4}{M_Z^4} - \frac{1}{24}\frac{M_H^2}{M_Z^2}\right) + \log M_Z^2\left(-\frac{1}{24}\frac{(M_H^2 - M_Z^2)}{M_Z^2}\right) \\
& -\frac{1}{24}\frac{M_H^4}{M_Z^4} + \frac{1}{12}\frac{M_H^2}{M_Z^2} + \frac{1}{9}c_W^2 - \frac{7}{72}\Big]
\end{aligned} \tag{321}$$

$$\begin{aligned}
\delta Z^{1/2}_{ZZ,fin,f} = \quad & \frac{\alpha}{4\pi s_W^2 c_W^2}\left(-\frac{1}{4}\right)\sum_f\Big[\Big\{(2I_3 - 4Q_f s_w^2)^2 + 1\Big\}\Big(-F_{12}(f,f,Z) - M_Z^2 G_{12}(f,f,Z)\Big) \\
& +m_f^2 G_0(f,f,Z)\Big]
\end{aligned} \tag{322}$$

$\underline{\delta Z_W^{1/2}}$

$$\delta Z_W^{1/2} = \frac{\alpha}{4\pi s_W^2}\left[\left(\frac{19}{12} - c_W^2\tilde{\beta} - s_W^2\tilde{\alpha} - \frac{1}{6}\sum_{doublet}1\right)C_{\mathrm{UV}}\right] + \delta Z^{1/2}_{W,fin,b} + \delta Z^{1/2}_{W,fin,f} \tag{323}$$

$$
\begin{aligned}
\delta Z^{1/2}_{W,fin,b} = \;& \frac{\alpha}{4\pi s_W^2}\Big[F_0(Z,W,W)\left(-2c_W^2 - \frac{1}{4c_W^2} - \frac{1}{24c_W^4} + \frac{2}{3} + c_W^2\tilde{\beta}\right) \\
& +G_0(Z,W,W)\left(-2c_W^2 + \frac{2}{3c_W^2} + \frac{1}{24c_W^4} - \frac{17}{6}\right)M_W^2 \\
& +F_0(H,W,W)\left(-\frac{1}{24}\frac{M_H^4}{M_W^4} + \frac{1}{12}\frac{M_H^2}{M_W^2}\right) \\
& +G_0(H,W,W)\left(+\frac{1}{24}\frac{M_H^4}{M_W^4} - \frac{1}{6}\frac{M_H^2}{M_W^2} + \frac{1}{2}\right)M_W^2 \\
& +F_0(A,W,W)\left(-2s_W^2 + s_W^2\tilde{\alpha}\right) + G_0(A,W,W)\left(-2s_W^2 M_W^2\right) \\
& +\log M_W^2\left(-\frac{1}{24}\frac{M_H^2}{M_W^2} - \frac{1}{24c_W^2} + \frac{1}{12}\right) \\
& +\log M_H^2\left(\frac{1}{24}\frac{M_H^4}{M_W^4} - \frac{1}{24}\frac{M_H^2}{M_W^2}\right) + \log M_Z^2\left(\frac{7}{24c_W^2} + \frac{1}{24c_W^4} - \frac{1}{3}\right) \\
& -\frac{1}{24}\frac{M_H^4}{M_W^4} + \frac{1}{12}\frac{M_H^2}{M_W^2} - \frac{1}{4c_W^2} - \frac{1}{24c_W^4} + \frac{11}{36}\Big]
\end{aligned}
\tag{324}
$$

$$
\begin{aligned}
\delta Z^{1/2}_{W,fin.f} = \;& \frac{\alpha}{4\pi s_W^2}\left(-\frac{1}{4}\right)\sum_{doublet}\Big[-4(F_{12}(f,f',W) + G_{12}(f,f',W)M_W^2) \\
& +2m_f^2 G_1(f',f,W) + 2m_f^{\prime 2} G_1(f,f',W)\Big]
\end{aligned}
\tag{325}
$$

<u>δM_Z^2</u>

$$
\delta M_Z^2 = \delta M_{Z,b}^2 + \delta M_{Z,f}^2 \tag{326}
$$

$$
\begin{aligned}
\delta M_{Z,b}^2 = \quad & \frac{\alpha}{4\pi s_W^2 c_W^2} M_Z^2 \Big[\left(-7c_W^4 + \frac{5}{3}c_W^2 + \frac{7}{6} \right) C_{\mathsf{UV}} \\
& + F_0(W,W,Z) \left(4c_W^6 + \frac{17}{3}c_W^4 - \frac{4}{3}c_W^2 - \frac{1}{12} \right) \\
& + F_0(H,Z,Z) \left(-\frac{1}{12}\frac{M_H^4}{M_Z^4} + \frac{1}{3}\frac{M_H^2}{M_Z^2} - 1 \right) \\
& + \log M_W^2 \left(-4c_W^6 + \frac{4}{3}c_W^4 - \frac{1}{3}c_W^2 \right) + \log M_H^2 \left(\frac{1}{12}\frac{M_H^4}{M_Z^4} - \frac{1}{4}\frac{M_H^2}{M_Z^2} \right) \\
& + \log M_Z^2 \left(-\frac{1}{12}\frac{M_H^2}{M_Z^2} - \frac{1}{12} \right) - \frac{1}{12}\frac{M_H^4}{M_Z^4} + \frac{1}{6}\frac{M_H^2}{M_Z^2} - \frac{2}{9}c_W^2 + \frac{1}{36} \Big]
\end{aligned} \tag{327}
$$

$$
\begin{aligned}
\delta M_{Z,f}^2 = \quad & \frac{\alpha}{4\pi s_W^2 c_W^2} M_Z^2 \left(+\frac{1}{2} \right) \sum_f \Big[\Big\{ (2I_3 - 4Q_f s_w^2)^2 + 1 \Big\} \left(\frac{1}{6} C_{\mathsf{UV}} - F_{12}(f,f,Z) \right) \\
& - \frac{m_f^2}{M_Z^2} \left(C_{\mathsf{UV}} - F_0(f,f,Z) \right) \Big]
\end{aligned} \tag{328}
$$

<u>δM_W^2</u>

$$
\delta M_W^2 = \delta M_{W,b}^2 + \delta M_{W,f}^2 \tag{329}
$$

$$
\begin{aligned}
\delta M_{W,b}^2 = \quad & \frac{\alpha}{4\pi s_W^2} M_W^2 \Big[\left(-\frac{31}{6} + \frac{1}{c_W^2} \right) C_{\mathsf{UV}} \\
& + F_0(Z,W,W) \left(4c_W^2 + \frac{17}{3} - \frac{4}{3c_W^2} - \frac{1}{12c_W^4} \right) \\
& + F_0(H,W,W) \left(-\frac{1}{12}\frac{M_H^4}{M_W^4} + \frac{1}{3}\frac{M_H^2}{M_W^2} - 1 \right) + F_0(A,W,W) \left(4s_W^2 \right) \\
& + \log M_W^2 \left(-\frac{1}{12}\frac{M_H^2}{M_W^2} - \frac{3}{2} - \frac{1}{12c_W^2} \right) + \log M_H^2 \left(\frac{1}{12}\frac{M_H^4}{M_W^4} - \frac{1}{4}\frac{M_H^2}{M_W^2} \right) \\
& + \log M_Z^2 \left(-2 + \frac{5}{12c_W^2} + \frac{1}{12c_W^4} \right) - \frac{1}{12}\frac{M_H^4}{M_W^4} + \frac{1}{6}\frac{M_H^2}{M_W^2} + \frac{7}{18} - \frac{1}{2c_W^2} - \frac{1}{12c_W^4} \Big]
\end{aligned} \tag{330}
$$

$$
\begin{aligned}
\delta M^2_{W,f} = \quad & \frac{\alpha}{4\pi s_W^2} M_W^2 \left(+\frac{1}{2}\right) \sum_{doublet} \Big[4 \left(\frac{1}{6} C_{\mathsf{UV}} - F_{12}(f,f',W)\right) \\
& - \frac{m_f^2 + m_f'^2}{M_W^2} C_{\mathsf{UV}} + \frac{2m_f^2}{M_W^2} F_1(f',f,W) + \frac{2m_f'^2}{M_W^2} F_1(f,f',W) \Big]
\end{aligned}
\tag{331}
$$

10.2 Higgs part

$\underline{\delta Z_H^{1/2}}$

$$
\delta Z_H^{1/2} = \frac{\alpha}{4\pi s_W^2} \left[\left(\frac{1}{4c_W^2} + \frac{1}{2} + \frac{1}{4c_W^2}\tilde{\varepsilon} + \frac{1}{2}\tilde{\delta} - \frac{1}{4} \sum_f \frac{m_f^2}{M_W^2} \right) C_{\mathsf{UV}} \right] + \delta Z^{1/2}_{H,fin,b} + \delta Z^{1/2}_{H,fin,f} \tag{332}
$$

$$
\begin{aligned}
\delta Z^{1/2}_{H,fin,b} = \quad & \frac{\alpha}{4\pi s_W^2} \Big[F_0(W,W,H) \left(-\frac{1}{2} - \frac{1}{2}\tilde{\delta}\right) \\
& + G_0(W,W,H) \left(\frac{1}{8} \frac{M_H^2}{M_W^2} - \frac{1}{2} + \frac{3}{2} \frac{M_W^2}{M_H^2} \right) M_H^2 \\
& F_0(Z,Z,H) \left(-\frac{1}{4c_W^2} - \frac{1}{4c_W^2}\tilde{\varepsilon} \right) \\
& + G_0(Z,Z,H) \left(\frac{1}{16} \frac{M_H^2}{M_W^2} - \frac{1}{4c_W^2} + \frac{3}{4c_W^2} \frac{M_Z^2}{M_H^2} \right) M_H^2 \\
& + G_0(H,H,H) \left(\frac{9}{16} \frac{M_H^2}{M_W^2} \right) M_H^2 \Big]
\end{aligned}
\tag{333}
$$

$$
\delta Z^{1/2}_{H,fin,f} = \frac{\alpha}{4\pi s_W^2} \sum_f \frac{m_f^2}{4M_W^2} \left[F_0(f,f,H) + M_H^2 \left(1 - \frac{4m_f^2}{M_H^2}\right) G_0(f,f,H) \right] \tag{334}
$$

$\underline{\delta M_H^2}$

$$
\delta M_H^2 = \delta M^2_{H,b} + \delta M^2_{H,f} \tag{335}
$$

$$
\begin{aligned}
\delta M^2_{H,b} = \; & \frac{\alpha}{4\pi s_W^2} M_H^2 \Big[\left(\frac{3}{4c_W^2} - \frac{3}{4}\frac{M_H^2}{M_W^2} + \frac{3}{2} \right) C_{\mathsf{UV}} \\
& + F_0(W,W,H) \left(-\frac{1}{4}\frac{M_H^2}{M_W^2} + 1 - 3\frac{M_W^2}{M_H^2} \right) \\
& + F_0(Z,Z,H) \left(-\frac{1}{8}\frac{M_H^2}{M_W^2} + \frac{1}{2}\frac{1}{c_W^2} - \frac{3}{2}\frac{M_Z^2}{c_W^2 M_H^2} \right) + F_0(H,H,H) \left(-\frac{9}{8}\frac{M_H^2}{M_W^2} \right) \\
& + \log M_W^2 \left(\frac{1}{2} + 3\frac{M_W^2}{M_H^2} \right) + \log M_H^2 \left(\frac{3}{4}\frac{M_H^2}{M_W^2} \right) \\
& + \log M_Z^2 \frac{1}{c_W^2} \left(\frac{1}{4} + \frac{3}{2}\frac{M_Z^2}{M_H^2} \right) - \frac{1}{4c_W^2} - \frac{3}{2c_W^2}\frac{M_Z^2}{M_H^2} - \frac{3}{4}\frac{M_H^2}{M_W^2} - \frac{1}{2} - 3\frac{M_W^2}{M_H^2} \Big]
\end{aligned} \tag{336}
$$

$$
\delta M^2_{H,f} = \frac{\alpha}{4\pi s_W^2} M_H^2 \sum_f \frac{m_f^2}{M_W^2} \left[\frac{1}{2} C_{\mathsf{UV}} + 2\frac{m_f^2}{M_H^2} \left(1 - \log m_f^2 \right) - \frac{1}{2} \left(1 - \frac{4m_f^2}{M_H^2} \right) F_0(f,f,H) \right] \tag{337}
$$

10.3 Charge renormalization

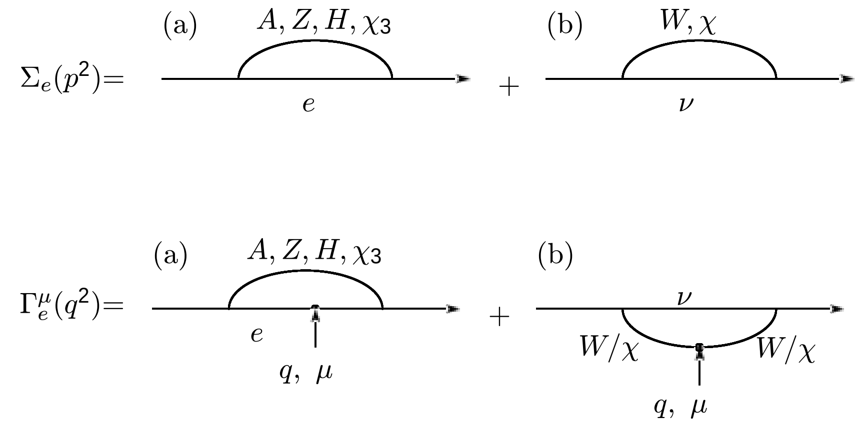


Figure 6: Electron self energy and eeA vertex

In order to calculate the charge renormalization constant δY, we calculate the eeA one-loop vertex $\Gamma_e^\mu(0)$. Here, the momentum of A is 0 and electrons are on mass-shell.

The diagrams are shown in Fig.6. Due to the identities

$$
\frac{\partial}{\partial p_\mu} \left(\frac{-1}{\not p + \not \ell - m} \right) = \frac{-1}{\not p + \not \ell - m} \gamma^\mu \frac{-1}{\not p + \not \ell - m} \tag{338}
$$

and

$$
\frac{\partial}{\partial p_\mu} \left(\frac{1}{(p+\ell)^2 - M^2} \right) = \frac{-2(p+\ell)^\mu}{((p+\ell)^2 - M^2)^2}, \tag{339}
$$

The major part of terms in $\Gamma_e^\mu(0)$ can be calculated by the corresponding fermion self energy. Using the notation in Fig.6,

$$(a)A, Z, H, \chi_3 \qquad \Gamma_e^\mu(0) = (-e)\,\frac{\partial}{\partial p_\mu}\Sigma(p^2)\Big|_{\not p = m}, \tag{340}$$

$$(b)W, \chi \qquad \Gamma_e^\mu(0) = (-e)\,\frac{\partial}{\partial p_\mu}\Sigma(p^2)\Big|_{\not p = m} + G_e^\mu. \tag{341}$$

We write the (b) terms for $\Gamma_e^\mu(0)$ explicitly.

$$e\left(\frac{e}{\sqrt{2}s_W}\right)^2 B\frac{-1}{\not p - \not \ell}A\frac{N}{(\ell^2 - M_W^2)^2} \tag{342}$$

$$\text{where} \quad \begin{cases} B = \gamma_\beta L, A = \gamma_\alpha L, N = 2g^{\alpha\beta}\ell^\mu - (1-\tilde\alpha)(\ell^\alpha g^{\beta\mu} + \ell^\beta g^{\alpha\mu}) & WW \\ B = (m_e/M_W)L, A = (m_e/M_W)R, N = -2\ell^\mu & \chi\chi \\ B = \gamma_\beta L, A = (m_e/M_W)R, N = (1-\tilde\alpha)M_W g^{\beta\mu} & W\chi \\ B = (m_e/M_W)L, A = \gamma_\alpha L, N = (1-\tilde\alpha)M_W g^{\alpha\mu} & \chi W \end{cases} \tag{343}$$

The terms proportional to $(1-\tilde\alpha)$ contribute to G_e^μ term.

$$\begin{aligned} G_e^\mu = \ & e\left(\frac{e}{\sqrt{2}s_W}\right)^2 (1-\tilde\alpha)\frac{d}{dM_W^2}\int\frac{d^n\ell}{i(2\pi)^n}\frac{1}{\ell^2 - M_W^2} \\ & \left[\gamma^\mu L\frac{1}{\not p - \not \ell}\not \ell L + \not \ell L\frac{1}{\not p - \not \ell}\gamma^\mu L - \gamma^\mu L\frac{1}{\not p - \not \ell}m_e R - m_e L\frac{1}{\not p - \not \ell}\gamma^\mu L\right] \end{aligned} \tag{344}$$

In the handling of the numerator, we can use the mass-shell condition to replace leftest(rightest) $\not p$ by m_e. After the integration by ℓ and differentiation by M_W^2,

$$G_e^\mu = \frac{e}{16\pi^2}\left(\frac{e}{\sqrt{2}s_W}\right)^2(1-\tilde\alpha)\int_0^1 dx\left[2x\left(C_{\mathsf{UV}} + \frac{1}{2} - \log D_2\right) + 2D_2\frac{-x}{D_2} + m_e^2x^2\frac{-x}{D_2}\right](-2\gamma^\mu L) \tag{345}$$

where $D_2 = xM_W^2 - x(1-x)m_e^2$. After the integral by x we obtain

$$G_e^\mu = -\frac{\alpha}{4\pi}\frac{e}{s_W^2}(1-\tilde\alpha)(C_{\mathsf{UV}} - \log M_W^2)\gamma^\mu L \tag{346}$$

Here it should be noted that the final result is independent of the electron mass m_e.

As defined in Sec.7.5,

$$\Sigma(p^2) = K_1 I + K_\gamma \not p + K_{5\gamma}\not p\gamma_5 \tag{347}$$

where we have dropped $K_5\gamma_5$. Then

$$\frac{\partial}{\partial p_\mu}\Sigma(p^2) = 2p^\mu(K_1 I + K_\gamma \not p + K_{5\gamma}\not p\gamma_5) + K_\gamma\gamma^\mu + K_{5\gamma}\gamma^\mu\gamma_5 \tag{348}$$

Using the identity $2p^\mu = \not p\gamma^\mu + \gamma^\mu \not p$ and the mass-shell condition, it becomes

$$\begin{aligned} \frac{\partial}{\partial p_\mu}\Sigma(p^2)\Big|_{\not p = m} &= (2m_eK_1'(m_e^2) + 2m_e^2K_\gamma'(m_e^2) + K_\gamma(m_e^2))\gamma^\mu + K_{5\gamma}(m_e^2)\gamma^\mu\gamma_5 \\ &= -2\delta Z_{eL}^{1/2}\gamma^\mu L - 2\delta Z_{eR}^{1/2}\gamma^\mu R \end{aligned} \tag{349}$$

Here Eq.224 is used. We obtain

$$\Gamma_e^\mu(0) = (-e)\left[-2\delta Z_{eL}^{1/2}\gamma^\mu L - 2\delta Z_{eR}^{1/2}\gamma^\mu R\right] + G_e^\mu \tag{350}$$

The counter term for eeA vertex, $\Gamma_e^\mu(0)$, is defined in Sec.7.4.8. The sum of loop term and the counter term is as follows.

$$\begin{aligned}\tilde{\Gamma}_e^\mu(0) &= \Gamma_e^\mu(0) + \hat{\Gamma}_e^\mu(0) \\ &= (-e\gamma^\mu)\left(\delta Y + \delta Z_{AA}^{1/2} - \frac{s_W}{c_W}\delta Z_{ZA}^{1/2}\right) + \left[-\frac{e}{2s_W c_W}\delta Z_{ZA}^{1/2}\gamma^\mu L + G_e^\mu\right]\end{aligned} \tag{351}$$

The second term ($[\cdots]$), which is proportional to $1-\tilde{\alpha}$ and $\gamma^\mu L$ becomes 0 by Eq.346 and Eq.317. By the renormalization condition (Eq.225) $\tilde{\Gamma}_e^\mu(0) = 0$,

$$\delta Y = -\delta Z_{AA}^{1/2} + \frac{s_W}{c_W}\delta Z_{ZA}^{1/2} \tag{352}$$

By Eq.316 and Eq.317,

$$\delta Y = \frac{\alpha}{4\pi}\left\{-\frac{7}{2}(C_{\mathsf{UV}} - \log M_W^2) - \frac{1}{3} + \frac{2}{3}\sum_f Q_f^2(C_{\mathsf{UV}} - \log m_f^2)\right\}. \tag{353}$$