Short Abstracts of "Introductory Workshop on Path Integrals and Pseudo-Differential Operators" at RIMS on October 7 - 10 in 2014

A integration by parts formula for Feynman path integrals

Daisuke Fujiwara (Gakushuin University, Japan)

We shall discuss Feynman path integral $\int_{\Omega_{x,y}} F(\gamma)e^{i\nu S(\gamma)}\mathcal{D}(\gamma)$ of a functional $F(\gamma)$ as was defined by Kumano-go. Here $\Omega_{x,y}$ is the set of paths $\gamma(t)$ in $\mathbf{R}^d$ such that $\gamma(0) = y, \gamma(T) = x$ and $S(\gamma)$ is the action of γ and $\nu = 2\pi h^{-1}$, with Planck's constant h. If $p(\gamma)$ is a vector field on $\Omega_{x,y}$ with suitable property, we prove the following integration by parts formula for Feynman path integral:

$$\begin{aligned}&\int_{\Omega_{x,y}} DF(\gamma)[p(\gamma)]e^{i\nu S(\gamma)}\mathcal{D}(\gamma)\\ &= -\int_{\Omega_{x,y}} F(\gamma)\mathrm{Div}\,p(\gamma)e^{i\nu S(\gamma)}\mathcal{D}(\gamma) - i\nu\int_{\Omega_{x,y}} F(\gamma)DS(\gamma)[p(\gamma)]e^{i\nu S(\gamma)}\mathcal{D}(\gamma).\end{aligned}$$

Here $DF(\gamma)[p(\gamma)]$ and $DS(\gamma)[p(\gamma)]$ are differentials of $F(\gamma)$ and $S(\gamma)$ evaluated in the direction of $p(\gamma)$, respectively, and $\mathrm{Div}\,p(\gamma)$ is divergence of vector fields $p(\gamma)$.

White noise approach to path integrals: From Lagrangian to Hamiltonian

Takeyuki Hida (Nagoya University, Japan)

In the quantum mechanics, one can see, around the classical path, there are many possible trajectories which, we propose, are expressed as sample paths of a Browninan bridge. With this idea we shall calculate the propagators, through which we can see the reason why non-commutativity appears.

Comparisons among simple formulas for generalized conditional Wiener integrals and its applications over continuous paths

Dong Hyun Cho (Kyonggi University, Korea)

Let $C[0,T]$ denote a generalized Wiener space, the space of real-valued continuous functions on the interval $[0,T]$ and define a stochastic process $Z : C[0,T] \times [0,T] \to \mathbf{R}$ by

$$Z(x,t) = \int_0^t h(u)dx(u) + x(0) + a(t)$$

for $x \in C[0,T]$ and $t \in [0,T]$, where $h \in L_2[0,T]$ with $h \neq 0$ a.e. and a is a continuous function on $[0,T]$. In this talk we introduce various simple formulas for generalized conditional Wiener integrals of functions on $C[0,T]$ with the conditioning functions having the drift a and investigate relationships among the simple formulas. As applications of the formulas we evaluate more generalized conditional Wiener integrals of functions on $C[0,T]$ with the drift which are of interests in the Feynman integration theories themselves and quantum mechanics.

Boundary value problems of differential equations with irregular singularities in microlocal analysis

Yasuo Chiba (Tokyo University of Technology, Japan)

In this talk, I will briefly give an introduction of boundary value problems from a microlocal point of view. By using microdifferential operators, the equation with irregular singularities can be transformed into the one with regular singularities. In particular, I will focus on the hyperbolic case. I shall present the properties of solutions for the boundary value problems as well as correspondence to the asymptotic solutions.

On the construction of the Feynman path integral for the Dirac equations

Wataru Ichinose (Shinshu University, Japan)

It has been considered for a long time that the simple construction of the Feynman path integral for the Dirac equations is impossible, as is written in the sections 2- 6 and etc. of the Feynman and Hibbs' book (1965). In this talk we show that the Feynman path integral for the Dirac equations can be given simply and mathematically. The Feynman path integral, which we give, is determined in the form of the sum-over-histories satisfying the superposition principle, i.e. the "sum" of the probability amplitudes with a common weight over all paths. We consider paths that go in any direction at any speed forward and also "backward" in time. We note that Feynman himself expected such paths to be considered.

Path integrals and stochastic analysis with Bernstein processes

Jean-Claude Zambrini (University of Lisbon, Portugal)

The talk will start with a review of the main interpretations of Feynman path integral. It will be indicated that his key concept is the one of "Transition element". An apparently unrelated problem of E. Schroedinger, dating back to 1931 will be described. Its solution involves a special class of stochastic processes described qualitatively by Bernstein. The solution of Schroedinger problem will be compared with Feynman's theory. Some examples will be considered. The stochastic deformation of Euler-Lagrange and Hamilton equations will be presented. A way to solve those equations will be described, inspired by Jacobi's classical integration method. Some new directions of research will conclude the presentation.

Feynman path integrals as infinite dimensional oscillatory integral

Sonia Mazzucchi (University of Trento, Italy)

In this talk I shall explain why the rigorous mathematical definition of Feynman path integrals requires the implementation of integration techniques on infinite dimensional spaces which go beyond measure theory and Lebesgue's "traditional" integration theory. I shall describe in details the theory of "infinite dimensional oscillatory integrals" and some of its applications, showing that they go beyond quantum mechanics and Schrödinger equation, but can be applied to more general dynamical systems.

On three imaginary-time path integral formulas with magnetic fields in relativistic quantum mechanics

Takashi Ichinose (Kanazawa University, Japan)

We consider three relativistic magnetic Schrödinger operators and give path integral representaions of the solutions of the corresponding imaginary-time Schrödinger equations.

Change of scale formulas for Wiener integrals

Byoung Soo Kim (Seoul National University of Technology, Korea)

It has long been known that Wiener measure and Wiener measurabil- ity behave badly under the change of scale transformation and under translations. Cameron and Storvick expressed the analytic Feynman integral on classical Wiener space as a limit of Wiener integrals. In doing so, they discovered nice change of scale formulas for Wiener integrals on classical Wiener space. In this talk we will introduce some spaces which are useful to study Wiener and Feynman integral, for example, classical Wiener space, abstract Wiener space, the space of abstract Wiener space valued continuous functions and a generalized func- tion space. We discuss change of scale formulas for Wiener integrals on these spaces. We also introduce change of scale formulas for Wiener integrals related with Fourier- Feynman transform and convolution product.

Applications of functional integrations to spectral analysis of QFT

Fumio Hiroshima (Kyushu University, Japan)

In the lecture we discuss a functional integral approach of the spectral analysis of a model in quantum field theory by means of Feynman-Kac type formula. It is demonstrated by using the so called Nelson model H which includes an linear interaction. 1)-3) are shown:
1) The bottom of the spectrum of H is an embedded eigenvalue and it is simple. The eigenvector associated with this is called the ground state.
2) The ground state has a spatially exponential decay and a gaussian decay in the field variable.
3) The Gibbs measure associated with the ground state is constructed and ground state expectation is expressed in terms of the Gibbs measure.
We also discuss the functional-integral-approach of some model with spin.

Phase space Feynman path integrals of higher order parabolic type

Naoto Kumano-go (Kogakuin University, Japan)

We give a general class of functionals for which the phase space path integrals of higher order parabolic type have a mathematically rigorous meaning. For any functional belonging to our class, the time slicing approximation of the phase space path integral converges uniformly on compact subsets with respect to the endpoint of position paths and to the starting point of momentum paths. Our class of functionals is rich because it is closed under addition and multiplication. The interchange of the order with the integration with respect to time and the interchange of the order with a limit hold in the phase space path integrals.