

RIMS Joint Research
Introductory Workshop on
Feynman Path Integral and Microlocal Analysis

Organizers: Naoto Kumano-go (Kogakuin University)
Byoung Soo Kim (Seoul National University of Science and Technology)
Susumu Yamazaki (Nihon University)
Yasuo Chiba (Tokyo University of Technology)

Date: from June 21 (Tuesday) to June 24 (Friday), 2011

Venue: RIMS, Kyoto University, Room No. 111

June 21 (Tuesday)

10:00~11:00 **Takashi Ichinose** (Kanazawa University)
Imaginary-time path integrals for three magnetic relativistic Schrödinger operators,
Part 1
11:20~12:20 **Bong Jin Kim** (Daejin University)
A note on the integral transforms on function spaces, Part 1
14:00~15:00 **Takeyuki Hida** (Nagoya University)
White noise approach to Feynman path integrals, Part 1
15:20~16:20 **Kun Sik Ryu** (Hannam University)
Introduction to the analogue of Wiener measure space and its applications, Part 1

June 22 (Wednesday)

10:00~11:00 **Daisuke Fujiwara** (Gakushuin University)
Stationary phase method for oscillatory integrals over a space of large dimension,
Part 1
11:20~12:20 **Bong Jin Kim** (Daejin University)
A note on the integral transforms on function spaces, Part 2

- 14:00~15:00 **Takeyuki Hida** (Nagoya University)
White noise approach to Feynman path integrals, Part 2
- 15:20~16:20 **Kun Sik Ryu** (Hannam University)
Introduction to the analogue of Wiener measure space and its applications, Part 2

June 23 (Thursday)

- 10:00~11:00 **Daisuke Fujiwara** (Gakushuin University)
Stationary phase method for oscillatory integrals over a space of large dimension,
Part 2
- 11:20~12:20 **Dong Hyun Cho** (Kyonggi University)
A survey of an analogue of conditional analytic Feynman integrals on a function space,
Part 1
- 14:00~15:00 **Byoung Soo Kim** (Seoul National University of Science and Technology)
Introduction to Feynman's operational calculi for noncommuting operators, Part 1
- 15:20~16:20 **Naoto Kumano-go** (Kogakuin University)
Phase space Feynman path integrals by time slicing approximation, Part 1

June 24 (Friday)

- 10:00~11:00 **Takashi Ichinose** (Kanazawa University)
Imaginary-time path integrals for three magnetic relativistic Schrödinger operators,
Part 2
- 11:20~12:20 **Dong Hyun Cho** (Kyonggi University)
A survey of an analogue of conditional analytic Feynman integrals on a function space,
Part 2
- 14:00~15:00 **Byoung Soo Kim** (Seoul National University of Science and Technology)
Introduction to Feynman's operational calculi for noncommuting operators, Part 2
- 15:20~16:20 **Naoto Kumano-go** (Kogakuin University)
Phase space Feynman path integrals by time slicing approximation, Part 2

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Kun Sik Ryu,

"Introduction to the analogue of Wiener measure space and its applications"

In this lecture, we will introduce the definition of analogue of Wiener measure space and the motivation of it. We investigate various properties of it - integration formulae for some functionals on analogue of Wiener space, the relationship among the Bartle integral and the measure-valued measure on analogue of Wiener space, the relationship among the Dovrakov integral and the operator-valued measure on analogue of Wiener space, the simple formula for conditional expectation with respect to analogue of Wiener measure, the measure-valued Feynman-Kac formula and Volterra integral equation, including the basic calculation for it.

Contents;

- 1) Historical Background and Preliminaries
- 2) The complex-valued analogue of Wiener measure
- 3) Fernique's Theorem and integration formula for some analogue of Wiener functionals
- 4) Translation Theorem and Paley-Wiener-Zygmund integral for analogue of Wiener measure
- 5) the relationship among the Bartle integral and the measure-valued measure on analogue of Wiener space
- 6) the relationship among the Dovrakov integral and the operator-valued measure on analogue of Wiener space
- 7) measure-valued Feynman-Kac formula
- 8) Volterra integral equation for the measure-valued Feynman-Kac formula.

Byoung Soo Kim,

"Introduction to Feynman's operational calculi for noncommuting operators"

Feynman's 1951 paper on the operational calculus for noncommuting operators arose out of his ingenious work on quantum electrodynamics and was inspired in part by his earlier work on the Feynman path integral. Indeed, Feynman thought of his operational calculus as a kind of generalized path integral. Much surprisingly varied work on the subject has been done since by mathematicians and physicists. Recently Jefferies and Johnson developed mathematical rigorous approach to Feynman's operational calculi. In this talk we give definitions and properties of Feynman's operational calculi initiated by Jefferies and Johnson. In particular, extraction of a linear factor and measure permutation formula for Feynman's operational calculi will be given.

Dong Hyun Cho,

"A survey of an analogue of conditional analytic Feynman integrals on a function space"

In this lecture, we introduce two kinds of simple formulas for the conditional expectations on the analogue of Wiener space which is introduced by Ryu and Im. Using the simple formulas, we evaluate the conditional analytic Feynman integrals of various kinds of functions which are useful in Feynman integration theories itself and quantum mechanics. We then find a solution of an integral equation which is formally equivalent to the Schrödinger differential equation. We also provide a change of scale transformation using the simple formulas and prove that the operator-valued Feynman integral can be expressed through the analogues of conditional analytic Feynman integrals on that space.