Multi-price lottery: A new pricing for ticket lottery sales

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Abstract. This paper proposes a multi-price lottery, a novel pricing mechanism for ticket lottery sales. The multi-price lottery improves the expected payoffs for all types of potential buyers compared to the traditional single-price lottery system, while also increasing the event organizer's overall sales. In this mechanism, high-paying buyers enjoy a higher probability of winning despite a higher price. Low-paying buyers benefit from lower prices despite a reduced chance of winning. Because some tickets are sold at higher prices, the organizer's total revenue increases even if the other tickets are sold at lower prices.
We begin by analyzing a simple case with two types of buyers—those with high and low valuations. We then extend the analysis to cases where buyer valuations are discrete and uniformly distributed. Finally, we examine the case with a continuous valuation distribution.

Keywords: Ticket lottery · Pricing mechanism · Expected payoff · Total revenue

1 Introduction

This paper proposes a multi-price lottery, a novel pricing mechanism for ticket lottery sales. The multi-price lottery improves the expected payoffs for all types of potential buyers compared to the traditional single-price lottery system, while also increasing the event organizer's overall sales.

The traditional ticket lottery sales are executed as follows. The event organizer decides the fixed price P and the number of tickets X. Potential buyers enter a lottery. Each lottery winner can buy one ticket for the price P. Though the lottery sales are simple and fair, some potential buyers are willing to pay higher prices to get tickets when the number of tickets is limited and the event is extremely popular. In this paper, we refer to potential buyers who can afford high prices as high-paying potential buyers. We refer to potential buyers who can only purchase the ticket at low prices as low-paying potential buyers.

The ticket prices of very popular events tend to be very expensive. Increasing the ticket price increases the event organizer's total sales and increases the possibility of winning for high-paying potential buyers, since low-paying potential buyers give up buying. The method's detriment is the dissatisfaction among low-paying potential buyers. They think that these events are only for the wealthy. Such an image is not good for some musicians and sports teams.

There are several other pricing methods, such as dynamic pricing[2, 6], auctions[1, 3], and a hybrid of auctions and lottery[4]. The ticket prices for popular events tend to become very expensive through these methods. These methods do not solve the low-paying potential buyers' problem.

There are other ticket lottery methods, such as weighted lottery[7], in which the potential buyers with some property have a higher winning probability. For a weighted lottery to be widely acceptable, all potential buyers need to be able to agree on how the weights are set. Such agreements might be difficult to reach among buyers who generally have differing values.

We propose a new ticket lottery in which the expected payoffs of the high-paying potential buyers and low-paying potential buyers are better than those of the usual ticket lottery sales, and the total sales of the organizer are also better than those of the usual ticket lottery sales.

The main idea is

- The winning probability of the high-paying potential buyers is increased, although the price is increased, and
- The price for the low-paying potential buyers is decreased, although the winning probability is decreased.

Because of the first property, the expected payoff of the high-paying potential buyers increases. Because of the second property, the expected payoff of the low-paying potential buyers also increases. Since some tickets are sold at a higher price, the total sales of the organizer increase even if some tickets are sold at a lower price. The proposed method can be used as a lottery sales method for fixed members, such as paid fan club members.

The rest of the paper is organized as follows. Section 2 discusses the simple case when there are two ticket valuations: high and low valuations. Section 3 considers maximizing the event organizer's total revenue. Section 4 shows variants of the multi-price lottery. Section 5 discusses the case when the valuations are discrete and uniform. Section 6 discusses the case when the valuation function is continuous. Section 7 concludes the paper.

2 High and low price case

Section 2-4 discusses the simplest case when there are two ticket valuations: high and low valuations. The assumption of the problem is as follows.

Assumption 1 *Let Y be the original price of the ticket. Let X be the number of tickets. Let $N(> X)$ be the total number of potential buyers. $r(0 < r < 1)$ is the ratio of high-paying potential buyers among N buyers. We assume that $rN > X$ and $(1 - r)N > X$. Let a_0Y and a_1Y $(a_0 > a_1 > 1)$ be the ticket valuation by the high-paying and the low-paying potential buyers, respectively.*

This paper assumes N is fixed, for example, the members of a musician's or a sports team's paid fan club. Y is the fixed value decided by the cost.

When the ticket price is Y, the expected payoff of a potential buyer whose ticket valuation is aY, and the possibility of getting the ticket by the lottery is p, the expected payoff for the potential buyer is $p(aY - Y)$.

When the standard lottery is executed, the possibility of winning is X/N for every potential buyer. Thus, the expected payoffs for the high-paying and low-paying potential buyers are $X(a_0 - 1)Y/N$ and $X(a_1 - 1)Y/N$, respectively. The total sales are XY.

Now, the organizer holds two different ticket lotteries. Each potential buyer may only enter one lottery. The two lotteries are low-price but low winning probability and high-price but high winning probability ones. The prices of the high-price and the low-price lotteries are b_0Y and b_1Y, respectively. Note that $b_0 > 1 > b_1$, $a_0 > b_0$, and $a_1 > b_1$ holds. The number of tickets sold by the high-price and the low-price lotteries is r_0X and $(1 - r_0)X$, respectively. Note that $r < r_0 < 1$ holds.

The expected payoff of the high-price lottery is greater than that of the standard lottery for a high-paying potential buyer if $r_0X(a_0 - b_0)Y/(rN) \geq X(a_0 - 1)Y/N$ holds. The inequality can be simplified as

$$r_0(a_0 - b_0)/r \geq a_0 - 1 \tag{1}$$

The expected payoff of the low-price lottery is greater than that of the standard lottery for a low-paying potential buyer if $(1 - r_0)X(a_1 - b_1)Y/((1 - r)N) \geq X(a_1 - 1)Y/N$ holds. The inequality can be simplified as

$$(1 - r_0)(a_1 - b_1)/(1 - r) \geq a_1 - 1 \tag{2}$$

To make the high-paying potential buyers join the high-price lottery, the expected payoff must be greater than that of the low-price lottery for a high-paying potential buyer. The condition is written as $r_0X(a_0 - b_0)Y/(rN) \geq (1 - r_0)X(a_0 - b_1)Y/((1 - r)N)$. Note that N is large, and one potential buyer's move does not affect the winning possibilities. The condition can be simplified as

$$r_0(a_0 - b_0)/r \geq (1 - r_0)(a_0 - b_1)/(1 - r) \tag{3}$$

To make the low-paying potential buyers join the low-price lottery, the expected payoff must be greater than that of the high-paying lottery for a low-paying potential buyer. The condition is written as $(1 - r_0)X(a_1 - b_1)Y/((1 - r)N) \geq r_0X(a_1 - b_0)Y/(rN)$. Again, we assume that one potential buyer's move does not affect the winning possibilities. The inequality can be simplified as

$$(1 - r_0)(a_1 - b_1)/(1 - r) \geq r_0(a_1 - b_0)/r \tag{4}$$

Note that if $b_0 > a_1$, this inequality is always satisfied because the term to the right of the inequality is negative.

The total sales of the new lotteries must be greater than those of the standard lottery. The condition is written as $r_0Xb_0Y + (1 - r_0)Xb_1Y \geq XY$ The condition can be simplified as

$$r_0b_0 + (1 - r_0)b_1 \geq 1 \tag{5}$$

For some values, there exists a pricing that satisfies all inequalities. Suppose that $a_0 = 10$, $a_1 = 1.5$ and $r = 0.6$. In this case, the organizer sets $b_0 = 2$, $b_1 = 0.5$, and $r_0 = 0.7$. Then all the inequalities (1)(2)(3)(4)(5) are strictly satisfied, that is, the expected payoff is better for high-paying and low-paying potential buyers than the ones of the standard lottery and the total sales are better than that of the standard lottery.

Generally, for any $\delta > 0$ that satisfies $\delta < 1 - r$, set $r_0 = r + \delta$, $b_0 = 1 + \delta(a_0 - 1)/(r + \delta)$, and $b_1 = 1 - \delta(a_1 - 1)/(1 - r - \delta)$, then the inequalities (3)(4)(5) are strictly satisfied and equalities are satisfied for (1)(2).

Theorem 1. *For any (a_0, a_1, r) and $\delta > 0$ that satisfies $\delta < 1 - r$, a multi-price lottery whose expected payoffs and total sales are better than those of a standard lottery exists when $r_0 = r + \delta$, $b_0 = 1 + \delta(a_0 - 1)/(r + \delta)$, and $b_1 = 1 - \delta(a_1 - 1)/(1 - r - \delta)$.*

When the event organizer knows a_0 and a_1, he/she can raise the price of the standard lottery up to $(1 - \epsilon)a_1 Y$ while still ensuring that all potential buyers participate. Even in this case, the above theorem holds for the new $a'_1 = 1/(1-\epsilon)$ and $a'_0 = a_0/(1 - \epsilon)a_1$.

The above discussion assumes that the event organizer correctly estimates every potential buyer's ticket valuation, and that there are two valuations. In many cases, the assumptions are not satisfied. Even in such a case, the above method can be applied if the potential buyers can be divided into a high-paying group and a low-paying group, M_0 and M_1. The lower bound estimate of M_0's potential buyer's ticket valuation is set to a_0. The upper bound estimate of M_1's potential buyer's ticket valuation is set to a_1. If the equations (1)(3) are satisfied by a_0, the equations are satisfied by any value $a' \geq a_0$, where a' is the ticket valuation of a potential buyer in M_0. If the equations (2)(4) are satisfied by a_1, the equations are satisfied by any value $a'' \leq a_1$, where a'' is the ticket valuation of a potential buyer in M_1.

Next, consider the case when the ratio of high-paying potential buyers r cannot be correctly estimated. The organizer evaluates that the ratio is between r_h and r_l, and the actual value r satisfies $r_l < r < r_h$. To satisfy equations (1)(3) in every case, r_h is used in these equations instead of r. To satisfy equations (2)(4) in every case, r_l is used in these equations instead of r.

Note that if we execute this lottery for general potential buyers, new low-paying potential buyers might appear when the ticket price decreases. Thus, this system works only for a fixed group of potential buyers, such as the members of a paid fan club. The design for the case when new potential buyers might join is an open problem.

Another problem with the proposed lottery is ticket resale by a low-paying potential buyer. Since some tickets are sold at a higher price, the resale price begins with the higher lottery price. That might result in a higher profit for the resellers. Again, one countermeasure to the problem is restricting the lottery to the paid fan club members. The members need to undergo identity verification during ticket sales and authentication during admission [5].

Note that some potential buyers may submit multiple applications for a single lottery sale to increase the winning probability. If every potential buyer submits the same number of applications, the result is unchanged. The case when each potential buyer submits a different number of applications is an open problem.

3 Maximizing the total sales

Since the organizer can decide the prices, he/she tries to maximize the total sales under the above conditions, that is, increase b_0 and b_1 if inequalities hold. Thus, the inequalities (1) and (2) are changed to the following equalities.

$$r_0(a_0 - b_0)/r = a_0 - 1. \tag{6}$$

$$(1 - r_0)(a_1 - b_1)/(1 - r) = a_1 - 1. \tag{7}$$

From these equations, we can obtain b_0 and b_1. Even if we use these values, inequalities (3)(4) are satisfied.

Using the values for b_0 and b_1, we obtain the total sales, the left side of equation (5)

$$(r_0 - r)(a_0 - a_1) + 1 \tag{8}$$

Since this equation has one variable r_0 and $a_0 > a_1$, the sales are maximized when r_0 is the maximum. When $r_0 = 1 - \epsilon$ (note that actually r_0 must be a little smaller than 1 to avoid $-\infty$ price for b_1), $b_0 = (1 - r/(1 - \epsilon))a_0 + r/(1 - \epsilon)$, and $b_1 = (1 - (1 - r)/\epsilon)a_1 + (1 - r)/\epsilon$.

When r_0 is large, $b_1 < 0$ might be satisfied. A negative price of b_1 can be achieved by giving the winners additional gifts other than the ticket. If the organizer wants to avoid a negative price, set $r_0 = 1 - (a_1 - 1)(1 - r)/a_1$. In this case, $b_1 = 0$ is achieved.

Another case to consider is that the number of high-paying potential buyers is small, and $rN < X$ holds. In this case, the winning probability of the high-paying potential buyers becomes 1 before r_0 becomes 1. In this case, the upper bound of r_0 is rN/X.

4 Variants of the multi-price lottery

Though the above lottery is simple, the high-paying winners might envy the low-paying winners. To avoid such envy, we can consider variants of the above multiple-price lottery. This section shows two variants of the lotteries from the previous section.

The first variant, called the multi-price multi-option lottery, is as follows. There are two options, and each potential buyer can apply for one option. The first option is joining the low-price lottery. The second option is joining the low-price lottery, and the losers must join the high-price lottery. The high-paying potential buyers are expected to select the second option.

Again, we set the prices of the high-price and the low-price lotteries as b_0Y and b_1Y, respectively. The number of tickets sold by the high-price and the low-price lotteries is r_0X and $(1-r_0)X$, respectively.

The expected payoff of the first option is greater than that of the standard lottery for the low-paying potential buyers if $(1-r_0)X(a_1-b_1)Y/N \geq X(a_1-1)Y/N$ holds. The inequality can be written as

$$(1-r_0)(a_1-b_1) \geq a_1 - 1. \tag{9}$$

Let p be the possibility of winning the low-price lottery. $p = (1-r_0)X/N$. The expected payoff of the second option is greater than the standard lottery for the high-paying potential buyers if $p(a_0-b_1)Y + (1-p)r_0X(a_0-b_0)Y/((1-p)rN) \geq X(a_0-1)Y/N$ holds. The inequality can be simplified as

$$(1-r_0)(a_0-b_1) + r_0(a_0-b_0)/r \geq a_0 - 1. \tag{10}$$

The high-paying potential buyers prefer the second option to the first option because the expected payoff of the first option is $p(a_0-b_1)Y$ and $a_0 > b_0$ holds. When the event organizer sets $b_0 > a_1$, the low-paying potential buyers prefer the first option to the second option. The total sales condition is the same as the original one

$$r_0b_0 + (1-r_0)b_1 \geq 1. \tag{11}$$

When $a_0 = 10$, $a_1 = 1.5$, and $r = 0.6$, the set of values of the previous section ($r_0 = 0.7$, $b_0 = 2$, and $b_1 = 0.5$) does not satisfy both of the inequalities (9) and (10). Inequalities (9)(10)(11) are strictly satisfied if $r_0 = 0.65$, $b_0 = 2$, and $b_1 = 0.05$.

Generally, for any $\delta > 0$ that satisfies $\delta < 1-r$, set $r_0 = r+\delta$, $b_0 = (1-r)a_0 + ra_1$, and $b_1 = 1-(a_1-1)(r+\delta)/(1-r-\delta)$, then $b_0 > a_1$ holds and inequality (11) is strictly satisfied and equalities are satisfied for (9)(10).

Theorem 2. *For any (a_0, a_1, r) and $\delta > 0$ that satisfies $\delta < 1-r$, a multi-price multi-option lottery whose expected payoffs and total sales are better than those of a standard lottery exists when $r_0 = r+\delta$, $b_0 = (1-r)a_0 + ra_1$, and $b_1 = 1-(a_1-1)(r+\delta)/(1-r-\delta)$.*

Again, consider maximizing the total sales. The inequalities in (9) and (10) are changed to the equalities as follows:

$$(1-r_0)(a_1-b_1) = a_1 - 1. \tag{12}$$

$$(1-r_0)(a_0-b_1) + r_0(a_0-b_0)/r = a_0 - 1. \tag{13}$$

From these equations, obtain b_0 and b_1. By substituting these equations into the left side of equation (11), we obtain the following equation that shows the total sales

$$r_0b_0 + (1-r_0)b_1 = r_0(1-r)(a_0-a_1) + 1 \tag{14}$$

The total sales are maximized when r_0 is maximized. Again, to avoid b_1's $-\infty$ price, $r_0 = 1-\epsilon$ and price $b_0 = (1-r)a_0 + ra_1$ and $b_1 = (1-(1-\epsilon)a_1)/\epsilon$. Another choice of pricing might be avoiding negative prices. In that case, $r_0 = 1/a_1$, $b_0 = (1-r)a_0 + ra_1$, and $b_1 = 0$.

The other variant of the multi-price lottery is called a multi-price multi-stage lottery. All potential buyers join the first(lowest price) lottery, and the losers can decide whether he/she continues to join the next lottery with a higher price. For the above two price valuation case, set variables r_0, b_0, and b_1 as the same ones for the multi-price multi-option lottery. Then, the inequalities to be satisfied are the same as the ones for the multi-price multi-option lottery (9)(10)(11).

Theorem 3. *For any (a_0, a_1, r) and $\delta > 0$ that satisfies $\delta < 1 - r$, a multi-price multi-stage lottery whose expected payoffs and total sales are better than those of a standard lottery exists when $r_0 = r + \delta$, $b_0 = (1-r)a_0 + ra_1$, and $b_1 = 1 - (a_1 - 1)(r + \delta)/(1 - r - \delta)$.*

5 Discrete uniform valuation case

This section discusses the case when the distribution of potential buyers' valuations is uniform but discrete.

Let $N = sn$ and the potential buyer $M_i (0 \le i \le n-1)$ has valuation $a_i Y$. $a_0 > a_1 > \cdots > a_{n-1}$ and $|M_i| = s$. $N > X$ is satisfied. Set $r = X/N$. Consider the case when the event organizer provides n lotteries. For some $k (0 < k < n-1)$, the event organizer sells at a higher price $b_i Y (b_i > 1)$ and a higher winning probability $r_i (r_i > r)$ for $M_i (i \le k)$ than the standard lottery . He/she sells at a lower price $b_i Y (b_i < 1)$ and a lower winning probability $r_i (r_i < r)$ for $M_i (i > k)$ than the standard lottery. Consider the case when each potential buyer can join one lottery, that is, the potential buyer in M_i joins the i-th lottery.

The conditions to be satisfied can be obtained as in the previous sections.

$$r_i(a_i - b_i) - r(a_i - 1) \ge 0 (0 \le i \le n-1) \tag{15}$$

$$r_i(a_i - b_i) - r_j(a_i - b_j) \ge 0 (j \ne i) \tag{16}$$

$$\sum_{i=0}^{n-1} r_i b_i - nr \ge 0 \tag{17}$$

If we want to maximize the total sales, we need to calculate optimal values that satisfy the equations (15)(16)(17). Obtaining the optimal values is not easy because of the many parameters.

The cases of a multi-option lottery and a multi-stage lottery can be similarly discussed.

6 Continuous distribution valuation case

This section discusses the case when the distribution of potential buyers' valuations is continuous. Let $f(x)$ be the potential buyers' distribution function. Let $a_0 Y$ and $a_1 Y$ be the maximum and minimum valuations. $\int_{a_1 Y}^{a_0 Y} f(x)dx = N$, where N is the total number of potential buyers. Consider the simple case when two lotteries L_0 and L_1 are provided. Let b_0 and $b_1 (b_0 \geq 1 \geq b_1)$ be the prices of the two lotteries. Let $aY(a_1 < a < a_0)$ be the potential buyer's border price to join L_0 or L_1. Let $r = \int_{aY}^{a_0 Y} f(x)dx/N$. r is the ratio of potential buyers whose valuation is greater than aY. $0 < r < 1$ holds. Let X be the number of tickets and let $r_0 (0 < r_0 < 1)$ be the ratio of the tickets sold by L_0.

The expected payoff of L_0 is greater than that of the standard lottery for a high-paying $(v > a)$ potential buyer if $r_0 X(vY - b_0 Y)/rN \geq X(vY - Y)/N$. The equation is simplified as

$$r_0(v - b_0)/r \geq v - 1 \tag{18}$$

The expected payoff of L_1 is greater than that of the standard lottery for a low-paying $(v < a)$ potential buyer if $(1 - r_0)X(vY - b_1 Y)/(1 - r)N \geq X(vY - Y)/N$. The equation is simplified as

$$(1 - r_0)(v - b_1)/(1 - r) \geq v - 1 \tag{19}$$

In equation (18)(19), set $v = a$ then we obtain the following two inequalities.

$$r_0(a - b_0)/r \geq a - 1 \tag{20}$$

$$(1 - r_0)(a - b_1)/(1 - r) \geq a - 1 \tag{21}$$

Since $b_0 \geq 1$, $r_0/r \geq 1$ holds from equation (20). Thus, $r_0 \geq r$ must be held. Therefore, $(1 - r_0) \leq (1 - r)$ is satisfied. Because $b_1 \leq 1$, equation (21) is satisfied only if $b_1 = 1$. In this case, $r_0 = r$ must also be satisfied. Therefore, no multi-price lottery satisfies all the conditions.

Note that for any multi-price lottery, there are two neighboring lotteries whose prices, b_0 and b_1, satisfy $b_0 \geq 1 \geq b_1$, the same argument holds. If the potential buyer's valuations are continuous, there does not exist a multi-price lottery that is better than the standard lottery.

By a similar discussion, we can show that there is no multi-option or multi-price lottery better than the standard lottery when the potential buyer's valuations are continuous.

7 Conclusion

This paper proposed a multi-price lottery, a novel pricing mechanism for ticket lottery sales. There are many remaining problems, for example,

- Comparison to the other pricing methods in real valuation distribution.
- Analysis when the potential buyers are not fixed in advance; that is, when a new lottery with a low price is presented, new potential buyers with lower valuations appear.
- A pricing method that maximizes the event organizer's total revenue for common discrete distributions of valuations.
- A new method that can treat continuous valuation functions of potential buyers.

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