THE TRANSACTIONS OF THE IEICE, VOL. E **71**, NO. **12** DECEMBER **1988**

PAPER ⟨Special Issue on CAS Karuizawa Workshop⟩

Reliable and Efficient Fixed Rountings on Digraphs

Yoshifumi MANABE†, Makoto IMASE†
and Terunao SONEOKA†, *Members*

SUMMARY The problem of constructing a reliable and efficient routing ρ in a communications network G is considered. The forwarding index $\xi(G, \rho)$, which is defined as the maximum number of routes which pass through each node, is a criterion of network efficiency. The diameter of the surviving route graph $D(R(G, \rho)/F)$, which is defined as the maximum number of surviving routes needed for communication between each pair of nodes if node and edge faults F occur, is a criterion of network reliability. Routings which minimize $\xi(G, \rho)$ and $D(R(G, \rho)/F)$ are needed. In this paper the following are shown: (1) A sufficient condition for k-connected digraphs ($k \neq 2, 4$) to have a routing ρ such that $D(R(G, \rho)/F) \leqq 6$ for $|F| < k$. (2) A method of constructing a digraph G and rounting ρ_2 such that $\xi(G, \rho_2) < 2\lceil \log_d n \rceil$ for any number of nodes n and maximum degree d. (3) A method of constructing a digraph G and routing ρ_2' such that $\xi(G, \rho_2) < 3\log_d n$ and $D(R(G, \rho_2')/F) \leqq 3$ for $|F| < d-1$ if $n > d^4$ and $d \geqq 3$.

1. Introduction

In the design of communications networks and multiprocessor networks, reliability and efficiency are two major factors. These networks are modeled by undirected graphs or directed graphs (digraphs), where nodes correspond to switching elements or processors, and edges correspond to communication links. The reliability and efficiency of networks are related to their topological properties (for example, connectivity and diameter) and a lot of graph-theoretical studies have been done, for example, Refs. (6), (8), (13) and Footnote*.

In the design of reliable and efficient networks, network control methods such as network routings must also be considered. Recently, two criteria for graphs and their routings have been proposed independently: the forwarding index for measuring efficiency[(3)], and the diameter of the surviving route graph for measuring reliability[(4),(2)].

A routing ρ assigns a fixed path between any pair of nodes, which is called a route. The forwarding index $\xi(G, \rho)$ is the maximum number of routes which pass through each node. No node can use its transmission capacity only for communications originating or terminating at it because it is also required to forward transmissions between other node pairs. If there is equal traffic between all pairs of node, the forwarding index shows the decrease in node capacity caused by forwarding. Thus, the forwarding index is a criterion of network efficiency.

The surviving route graph $R(G, \rho)/F$ is defined for a graph $G=(V, E)$, a routing ρ and a faulty component set F. Its nodes are $V-F$ and there is an edge from node u to node v in $R(G, \rho)/F$ wherever the route from u to v avoids F in G. The routing table is assumed to be computed only once for a given communications network configuration, and thus all messages must be sent by these routes. When a node or an edge fails, routes passing through it become unusable. However, communication is still possible through a sequence of surviving routes. The time required to send a message along a route is often dominated by the message processing time at the two terminal nodes of the route. Under this assumption, the diameter of the surviving route graph $D(R(G, \rho)/F)$ is a criterion of network reliability.

Towards attaining a good routing which minimizes both the forwarding index and the diameter of the surviving route graph, the following two problems have been considered[(10)].

(1) Construction of a good routing for the above criteria on general graphs.

(2) Construction of a good graph and routing pair for these criteria.

For problem (1) concerning the surviving route graph, it has been shown that if the routing is a minimal routing, such that the route from u to v is a minimal length path from u to v, the diameter of the surviving route graph depends on the number of faults[(4)]. Previous papers have shown sufficient conditions that k-connected undirected graphs have a nonminimal routing ρ such that $D(R(G, \rho)/F)$ is a small constant independent of the number of faults[(11),(7),(9)].

For problem (2) concerning the forwarding index, a method of constructing an undirected graph G and its routing ρ such that $\xi(G, \rho)$ has the same order as the lower bound is shown[(3)].

Manuscript received July 6, 1988.
† The authors are with NTT Software Laboratories, Musashino-shi, 180 Japan.

* T. Soneoka, H. Nakada and M. Imase: "A design of d-connected digraph with minimum number of edges and quasiminimal diameter I", submitted for Discrete Applied Mathematics.

In this paper, these problems are considered for the digraph case. For problem (1), Sect. 3 proposes a sufficient condition for k-connected digraphs ($k \neq 2, 4$) to have a routing ρ such that $D(R(G, \rho)/F) \leqq 6$ for $|F| < k$.

For problem (2), Sect. 4 shows two methods of constructing an n-node digraph G with the maximum degree d, and a routing $\rho_i (i=1, 2)$ whose forwarding index is small: ρ_1 such that $\xi(G, \rho_1)$ is asymptotically optimal if n and d are relatively prime, and ρ_2 such that $\xi(G, \rho_2)$ is about twice that of the lower bound for any n and d.

Previous work has considered either the forwarding index or the diameter of the surviving route graph. Section 5 shows two methods of constructing an n-node digraph G with the maximum degree d and routing such that both the forwarding index and the diameter of the surviving route graph are quasi-optimal. If n and d are relatively prime, $n > d^4$, and $d \geqq 3$, there is an n-node digraph G with the maximum degree d and routing ρ_1' such that $\xi(G, \rho_1')$ is at most twice that of the lower bound and $D(R(G, \rho_1')/F) \leqq 3$ for $|F| < d-1$. If $n > d^4$ and $d \geqq 3$, there is an n-node digraph G with the maximum degree d and routing ρ_2' such that $\xi(G, \rho_2')$ is at most 3 times that of the lower bound and $D(R(G, \rho_2')/F) \leqq 3$ for $|F| < d-1$.

2. Preliminary

2.1 Definitions

This section presents definitions and terminology. Let gcd (n, d) be the greatest common divisor for n and d. We define $f(n) = o(1)$ to mean $\lim_{n \to \infty} f(n) = 0$. Let $G = (V, E)$ be a directed graph where V is a set of nodes and E is a set of directed edges. Hereafter, we refer to directed graphs as digraphs. The outdegree or indegree of node v is the number of edges which are incident out of or into v. The maximum degree $\Delta(G)$ of a graph G is the largest value among the indegrees and outdegrees of all v in $V(G)$. For a node set $U \subset V$, the subgraph induced by U is the maximal subgraph of G with node set U.

If $(u, v) \in E$, then u is a predecessor of v; similarly, v is a successor of u. A walk from node v_0 to node v_k in G is an alternating sequence of nodes and edges, say $v_0, e_1, v_1, \cdots, v_{i-1}, e_i, v_i, \cdots, e_k, v_k$, where $e_i = (v_{i-1}, v_i) \in E$. A walk whose nodes are distinct is called a path. The distance from node u to node v, denoted by dis (u, v), is the length, or the number of edges, of the shortest walk from u to v. The diameter of G, $D(G)$, is the maximum distance between any pair of nodes.

Digraph G is said to be strongly connected if there exists a walk between every pair of distinct nodes. Digraph G is said to be k-connected if G remains strongly connected when any $k-1$ nodes are removed. The connectivity $\kappa(G)$ of digraph G is defined as the minimum number of nodes whose removal results in a trivial or not strongly connected digraph. For digraph G with $\Delta(G) = d$, $\kappa(G) \leqq d$.

For a node $v \in V$ and a node set $U \subset V - \{v\}$, v-U fan is a set of $|U|$ disjoint paths from v to all nodes of U. U-v fan is a set of $|U|$ disjoint paths from all nodes of U to v. The following property holds for k-connected digraphs.

[Property 1][1] Let $G = (V, E)$ be a k-connected digraph, U be any node subset of V such that $|U| \leqq k$, and v be any node in $V - U$. There are a v-U fan and a U-v fan. □

We refer the reader to Ref. (5) for the other graph terminology.

2.2 Forwarding Index and Diameter of Surviving Route Graph

For a network $G = (V, E)$, a routing ρ is a function that assigns a fixed path to an ordered pair (x, y) in $V \times V$. The path specified by $\rho(x, y)$ is called a route from x to y.

For node v in G and routing ρ, let $\xi_v(G, \rho)$ be the number of routes of ρ that pass through node v. The forwarding index, $\xi(G, \rho)$ is defined as

$$\xi(G, \rho) = \max_{v \in V(G)} \xi_v(G, \rho).$$

For the graph shown in Fig. 1 (a), an example of routing ρ is defined as follows.

$$\rho(u, v) = \begin{cases} u, (u, v), v & \text{if } (u, v) \in E \\ 0, (0, 1), 1, (1, 2), 2 & u=0, v=2 \\ 2, (2, 3), 3, (3, 0), 0, (0, 1), 1 & u=2, v=1 \\ 3, (3, 0), 0, (0, 1), 1 & u=3, v=1 \end{cases} \quad (1)$$

For the routing ρ on G, $\xi_0(G, \rho) = 2$, $\xi_1 = (G, \rho) = 1$, $\xi_2(G, \rho) = 0$, $\xi_3(G, \rho) = 1$, and $\xi(G, \rho) = 2$. For analyzing the minimum achievable forwarding index, we define the following terminology.

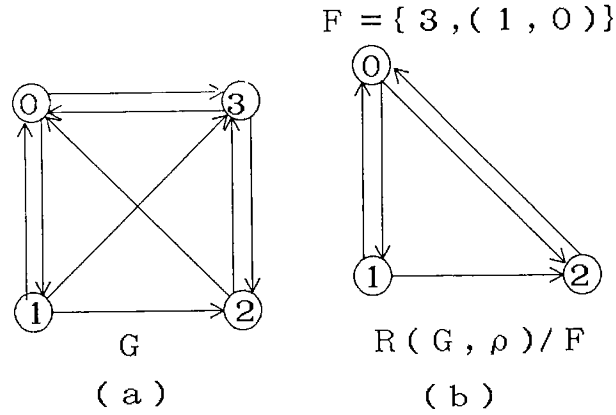


Fig. 1 An example of G and $R(G, \rho)/F$.

$$\xi(G)=\min_{\rho}\xi(G,\rho)$$

A lower bound on $\xi(G)$ can be obtained by an argument similar to that for undirected graph G[3].

[Property 2] For any digraph $G(n, d)$ with $|V(G)|=n$ and $\Delta(G)=d$,

$$\xi(G(n,d)\geqq(1+o(1))n\log_d n. \qquad \square$$

It is easily shown that digraphs which can achieve this lower bound must have a diameter of $O(\log_d n)$.

Let $F\subset V\cup E$ be a faulty component set, which consists of the faulty node set, F_v, and the faulty edge set, F_e. The surviving route graph, $R(G,\rho)/F=(V',E')$, is a digraph defined as follows.

$$V'=V-E_v,$$

$$E'=\{(x,y)|\,V(\rho(x,y))\cap F_v=\phi$$

$$\text{and } E(\rho(x,y))\cap F_e=\phi\},$$

where $V(\rho(x,y))$ and $E(\rho(x,y))$ are respectively the set of nodes and edges contained in route $\rho(x,y)$. The diameter of the surviving route graph is denoted by $D(R(G,\rho)/F)$. For the graph G in Fig. 1(a), its routing ρ defined by Eq. (1) and $F=\{3,(1,0)\}$, the surviving route graph $R(G,\rho)/F$ is shown in Fig. 1(b), and its diameter $D(R(G,\rho)/F)=2$. The lower bound of the diameter of the surviving route graph is given as follows.

[Property 3] For any digraph G and any routing ρ on it,

$$\max_{|F|<\kappa(G)} D(R(G,\rho)/F)\geqq 2. \qquad \square$$

If $|F|\geqq\kappa(G)$, G/F might be disconnected and $\max D(R(G,\rho)/F)$ is infinity. Therefore, digraphs which can have an optimal diameter of the surviving route graph must be maximally connected.

3. Construction of a Reliable Routing on General Networks

This section gives in Theorem 1 a sufficient condition for k-connected digraphs ($k\neq 2,4$) to have a routing ρ such that the diameter of the surviving route graph is a small constant. First, we define a condition $DC_k(m)$. This condition is similar to that defined in Refs. (11), (9) for undirected graphs. For $v\in V$, let $P_k(v)(S_k(v))$ be a set of v and $k-1$ predecessors (successors) of v. An example of $P_k(v)$ and $S_k(v)$ is shown in Fig. 2. Define $\Gamma_k(v)$ to be $P_k(v)\cup S_k(v)$.

[Definition 1] Condition $DC_k(m)$: $G=(V,E)$ has m nodes $v_0, v_1, \cdots, v_m$ such that there exists some pairs $P_k(v_i)$ and $S_k(v_i)(i=1,\cdots,m)$ which satisfies $\Gamma_k(v_i)\cap\Gamma_k(v_j)=\phi$ for any $i\neq j$. $\square$

[Theorem 1] If a k-connected ($k\neq 2,4$) digraph G satisfies the condition $DC_k(k)$, there is a routing ρ on G such that

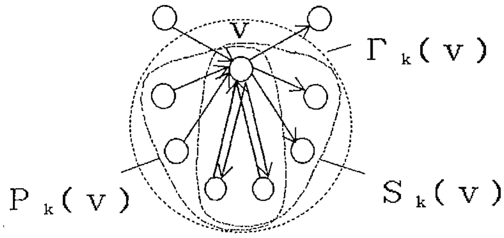


Fig. 2 $P_k(v)$, $S_k(v)$ and $\Gamma_k(v)(k=5)$.

$$\max_{|F|<k} D(R(G,\rho)/F)\leqq 6. \qquad \square$$

By a similar argument to that in Ref. (11), a class of digraphs satisfying $DC_k(k)$ is derived.

[Property 4] For every $0<\varepsilon<4^{-1/3}$, there exists an $n_0>0$ such that every k-connected digraph G with $n\geqq n_0$ nodes and $\Delta(G)\leqq\varepsilon\cdot n^{1/3}$ satisfies $DC_k(k)$. $\square$

Consequently, all graphs with such density have the routings described above.

Now define the routing ρ. Let $v_i(i=0,1,\cdots,k-1)$ be the set of nodes in $V(G)$ which attains the condition $DC_k(k)$. Define $\Gamma=\bigcup_{0\leqq i<k}\Gamma_k(v_i)$. Let $G(\Gamma_k(v))$ be the subgraph induced by $\Gamma_k(v)$.

From $|P_k(v_i)|=k$ ($|S_k(v_i)|=k$) and Property 1, there is a $u-P_k(v_i)$ fan (a $S_k(v_i)-u$ fan) for any $P_k(v_i)(S_k(v_i))$ and $u\notin P_k(v_i)(u\notin S_k(v_i))$, where $i=0,1,\cdots,k-1$. The path from $u\notin P_k(v_i)$ to $v\in P_k(v_i)$ contained in the $u-P_k(v_i)$ fan is denoted by $\Psi_{\text{fan}}(u,v;P_k(v_i))$. The path from $u\in S_k(v_i)$ to $v\notin S_k(v_i)$ contained in the $S_k(v_i)-v$ fan in denoted by $\Psi_{\text{fan}}(u;S_k(v_i),v)$. For any $u\in P_k(v_i)$ and $v\in S_k(v_i)$, there is a path from u to v in $G(\Gamma_k(v_i))$. This path is denoted by $\Psi_c(u,v;\Gamma_k(v_i))$.

Let u_i and u_j respectively be nodes in $S_k(v_i)$ and $P_k(v_j)(i\neq j)$. If the paths of a $u_i-P_k(v_j)$ fan are assigned to the routes from u_i to $P_k(v_j)$, it may be impossible to choose paths of a $S_k(v_i)-u_j$ fan as the routes from $S_k(v_i)$ to u_j, because the path from u_i to u_j in the $u_i-P_k(v_j)$ fan may not be the same as that in the $S_k(v_i)-u_j$ fan. In order to determine which of the $u_i-P_k(v_j)$ or $S_k(v_i)-u_j$ fans should be chosen as routes, we consider a digraph $T(k)=(V_T,E_T)$, where a node i in $V_T(i=0,1,\cdots,k-1)$ corresponds each $\Gamma_k(v_i)$, and if $(i_1,i_2)\in E_T$, then $(i_2,i_1)\notin E_T$.

Let u and v be arbitrary nodes in V. According to $T(k)$, a routing ρ can now be defined as follows:

$$\rho(u,v)=\begin{cases}\Psi_{\text{fan}}(u,v;P_k(v_i)) & \text{if } u\in V-\Gamma \text{ and } v\in P_k(v_i) \quad (2)\\ \Psi_{\text{fan}}(u;S_k(v_i),v) & \text{if } u\in S_k(v_i) \text{ and } v\in V-\Gamma \quad (3)\\ \Psi_{\text{fan}}(u,v;P_k(v_j)) & \text{if } u\in\Gamma_k(v_i) \text{ and } v\in P_k(v_j) \text{ such that } (i,j)\in E_T \quad (4)\\ \Psi_{\text{fan}}(u;S_k(v_j),v) & \text{if } u\in S_k(v_j) \text{ and } v\in\Gamma_k(v_i) \text{ such that } (i,j)\in E_T \quad (5)\end{cases}$$

$$\begin{cases} \Psi_{\rm fan}(u\,;\,S_k(v_i),v) & \text{if } u\in S_k(v_i) \text{ and} \\ & v\in \Gamma_k(v_i)-S_k(v_i) \quad (6) \\ \Psi_c(u,v\,;\,\Gamma_k(v_i)) & \text{if } u\in P_k(v_i) \text{ and } v\in S_k(v_i) \quad (7) \\ \text{don't care} & \text{otherwise} \end{cases}$$

It is easily checked that ρ is well defined, that is, at most one route is defined for each pair of nodes. The routing ρ satisfies the following property:

[Property 5] For any faulty set F with $|F|<k$,

$$D(R(G,\rho)/F)\leqq 2D(T(k))+2. \qquad \square$$

(Proof) Since $|F|<k$ and the number of node sets $\Gamma_k(v_i)$ is k, there exists a node set, $\Gamma_k(v_I)$ such that $G(\Gamma_k(v_I))$ contains no faulty component. Let u and v be arbitrary non-faulty distinct nodes in $V-F$ and let $\mathrm{dis}_R(u,v)$ be the distance from u to v in $R(G,\rho)/F$.

To prove $\mathrm{dis}_R(u,v)\leqq 2D(T(k))+2$, the following lemmas are used.

[Lemma 1] For any nodes $u\in P_k(v_I)$ and $v\in S_k(v_I)$, $\mathrm{dis}_R(u,v)\leqq 1$.

[Lemma 2] For any node $u\in V-\Gamma$, there is a node $v\in P_k(v_I)$ such that $\mathrm{dis}_R(u,v)=1$.

[Lemma 3] For any node $u\in\Gamma_k(v_i)(i\neq I)$, there is a node $v\in P_k(v_I)$ such that $\mathrm{dis}_R(u,v)\leqq D(T(k))$.

[Lemma 4] For any node $u\in S_k(v_I)$, there is a node $v\in P_k(v_I)$ such that $\mathrm{dis}_R(u,v)\leqq D(T(k))+1$.

[Lemma 5] For any node $v\in V-\Gamma$, there is a node $u\in S_k(v_I)$ such that $\mathrm{dis}_R(u,v)=1$.

[Lemma 6] For any node $v\in\Gamma_k(v_i)(i\neq I)$, there is a node $u\in S_k(v_I)$ such that $\mathrm{dis}_R(u,v)\leqq D(T(k))$.

[Lemma 7] For any node $v\in P_k(v_I)$, there is a node $u\in S_k(v_I)$ such that $\mathrm{dis}_R(u,v)\leqq 1$.

Using these Lemmas, $\mathrm{dis}_R(u,v)\leqq 2D(T(k))+2$ for any u, v is proved as follows:

[Case 1] Suppose $u\in P_k(v_I)$ and $v\in S_k(v_I)$. This case is proved in Lemma 1.

[Case 2] Suppose $u\notin P_k(v_I)$ and $v\in S_k(v_I)$. From Lemma 2, 3 and 4, there is a node $u'\in P_k(v_I)$ such that $\mathrm{dis}_R(u,u')\leqq D(T(k))+1$. From Lemma 1, $\mathrm{dis}_R(u',v)\leqq 1$. Thus, $\mathrm{dis}_R(u,v)\leqq D(T(k))+2$.

[Case 3] Suppose $u\in P_k(v_I)$ and $v\notin S_k(v_I)$. From Lemma 5, 6 and 7, there is a node $v'\in S_k(v_I)$ such that $\mathrm{dis}_R(v',v)\leqq D(T(k))$. From Lemma 1, $\mathrm{dis}_R(u,v')\leqq 1$. Thus, $\mathrm{dis}_R(u,v)\leqq D(T(k))+1$.

[Case 4] Suppose $u\notin P_k(v_I)$ and $v\notin S_k(v_I)$. From Lemma 2, 3 and 4, there is a node $u'\in P_k(v_I)$ such that $\mathrm{dis}_R(u,u')\leqq D(T(k))+1$. From Lemma 5, 6 and 7, there is a node $v'\in S_k(v_I)$ such that $\mathrm{dis}_R(v',v)\leqq D(T(k))$. From Lemma 1, $\mathrm{dis}_R(u',v')\leqq 1$. Thus, $\mathrm{dis}_R(u,v)\leqq 2D(T(k))+2$.

Therefore, for any u and v, $\mathrm{dis}_R(u,v)\leqq 2D(T(k))+2$. Now let us prove Lemma 1-7.

(Proof of Lemma 1) Since $G(\Gamma_k(v_I))$ contains no faulty component, it is trivial from Eq. (7).

(Proof of Lemma 2) From Eq. (2), there are k disjoint routes from u to $P_k(v_I)$. Since $|F|<k$, at least one of these is surviving, that is, there is a node $v\in P_k(v_I)$ such that $\mathrm{dis}_R(u,v)=1$.

(Proof of Lemma 3) Let $i_0(=i), i_1, i_2, \cdots, i_{d-1}, i_d(=I)$ be the node sequence of a path from i to I in $T(k)$ such that $d\leqq D(T(k))$. At least one of the k routes from u to $P_k(v_{i_1})$ is surviving in $G-F$ from Eq. (4). Let u_{i_1} be the terminal node in $P_k(v_{i_1})$ of this route, thus $\mathrm{dis}_R(u,u_{i_1})=1$. By iterating this discussion, it can be shown that there is a node $u_{i_{j+1}}\in P_k(v_{i_{j+1}})$ such that $\mathrm{dis}_R(u_{i_j},u_{i_{j+1}})=1$ for $j=0,1,\cdots,d-1$. Consequently, there is a node $v=u_{i_d}\in P_k(v_I)(I=i_d)$ such that $\mathrm{dis}_R(u,v)\leqq d\leqq D(T(k))$.

(Proof of Lemma 4) In $T(k)$, there is a walk from I to I with a distance not greater than $D(T(k))+1$, because there is a node $i\in V_T$ such that $(I,i)\in E_T$ and there is a path from i to I with a distance not greater than $D(T(k))$. In the same way as in the proof of Lemma 3, there is a node $v\in P_k(v_I)$ such that $\mathrm{dis}_R(u,v)\leqq D(T(k))+1$.

(Proof of Lemma 5) Similar to the proof of Lemma 2, by replacing Eq. (2) with Eq. (3).

(Proof of Lemma 6) There is a path from I to i in $T(k)$ with a distance not graeter than $D(T(k))$. Thus, it is clear by a similar argument to that in Lemma 3.

(Proof of Lemma 7) If $v\in S_k(v_I)\cap P_k(v_I)$, let $u=v$ and $\mathrm{dis}_R(u,v)=0$. If $v\in P_k(v_I)-S_k(v_I)$, from Eq. (6), there is a node $u\in S_k(v_I)$ such that $\mathrm{dis}_R(u,v)=1$. $\square$

It has been shown in Ref. (7) that the diameters of the following digraphs $T_1(k)$ and $T_2(k)$ are not larger than 2. Let m be $\lfloor k/2\rfloor$.

When k is odd, $T_1(k)=(V_{T_1},E_{T_1})$ is defined by $V_{T_1}=\{0,1,\cdots,2m\}$, $E_{T_1}=\bigcup_{\alpha=1}^{m}E_{T_1,\alpha}$.

Where $E_{T_1,\alpha}=\{(i,j)|j\equiv i+\alpha \pmod{2m+1}\}$.

When k is even and $k\neq 2,4$, $T_2(k)=V_{T_2},(E_{T_2})$ is defined by

$$V_{T_2}=\{0,1,\cdots,2m-1\},\ E_{T_2}=\bigcup_{\alpha=1}^{m}E_{T_2,\alpha}$$

where $E_{T_2,\alpha}=$

$$\begin{cases} \{(i,j)|j\equiv i+\alpha \pmod{2m}\} & \text{if } \alpha=1,3,4,\cdots,m-1 \\ \{(i,j)|i \text{ is odd and } j\equiv i+2 \pmod{2m}\} & \\ \quad\cup\{(i,j)|i \text{ is even and } j\equiv i-2 \pmod{2m}\} & \\ \qquad \text{if } \alpha=2 & \\ \{(i,j)|0\leqq i<m \text{ and } j\equiv i+m \pmod{2m}\} & \\ \qquad \text{if } \alpha=m & \end{cases}$$

From $D(T_i(k))\leqq 2(i=1,2)$ and Property 5, it is deduced that Theorem 1 is valid.

4. Construction of a Digraph with an Efficient Routing

This section gives a construction method of a digraph G and its routing ρ which has a quasi-optimal forwarding index. As shown is Sect. 2, digraphs which

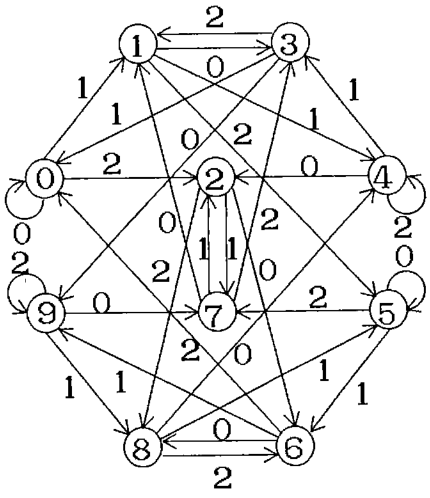


Fig. 3 $G_B(10, 3)$.

can have an optimal routing in terms of the forwarding index must have a diameter of $O(\log_d n)$. The generalized de Bruijn digraph[6] $G_B(n, d)$ has a $O(\log_d n)$ diameter. $G_B(n, d)=(V, E)$ is defined as follows.

$$V=\{0, 1, \cdots, n-1\}$$

$$E=\{(u, v)|v\equiv d\cdot u+p \pmod n),\ p=0, 1, \cdots, d-1\}$$

$G_B(10, 3)$ is shown in Fig. 3. It has been shown that $\Delta(G_B(n, d))=d$, $\kappa(G_B(n, d))=d-1$ and $D(G_B(n, d))=\lceil\log_d n\rceil$[8]. Let $D(G_B(n, d))$ be D. Thus, $d^{D-1}<n\leqq d^D$.

For preparation, we give a method of representing walks on $G_B(n, d)$. There is an m-length walk from node u to node v if and only if there are integers $p_1, p_2, \cdots, p_m$ satisfying $0\leqq p_i<d$ $(i=1, 2, \cdots, m)$ and

$$v\equiv u\cdot d^m+p_1\cdot d^{m-1}+p_2\cdot d^{m-2}+\cdots+p_{m-1}\cdot d+p_m$$

(mod n). This walk can be represented by u and the d-ary m digits $(p_1, p_2, \cdots, p_m)$. For example, since $1\equiv 0\cdot 3^3+1\cdot 3^2+0.3+2 \pmod{10}$, there is a 3-length walk from 0 to 1 in $G_B(10, 3)$: 0, (0, 1), 1, (1, 3), 3, (3, 1), 1. This walk is represented by 0 and (1, 0, 2).

Therefore, if there exists an integer $x(0\leqq x<d^m)$ satisfying

$$v\equiv u\cdot d^m+x \pmod n, \qquad (8)$$

there is an m-length walk from u to v, which is represented by u and d-ary m digit representation of x. Generally, this method gives walks rather than paths. However, paths are derived by shortening there walks when the same node appears more than once. Thus, we need only consider walks. From the definition of diameter, there doesn't always exist a walk of length $m<D$ from u to v. Let minimum nonnegetive integer x satisfying Eq. (8) be $P_m(u, v)$. If $P_m(u, v)<d^m$, let the walk derived from $P_m(u, v)$ be $W_m(u, v)$, and the set of routes $W_m(u, v)$ for all v be $W_m(u)$. There might be plural D-length walks because $P_D(u, v)+jn$ might be less than d^D for some $j>0$. Let $W_{D,j}(u, v)$ be the route derived from $P_D(u, v)+jn$ and the set of routes $W_{D,j}(u, v)$ for all v be $W_{D,j}(u)$. Note that $W_{D,0}(u, v)=W_D(u, v)$. Now a routing ρ_1 on $G_B(n, d)$ is defined as follows.

[Definition 2]

$$\rho_1(u, v)=W_{D,0}(u, v). \qquad \square$$

If $\gcd(n, d)=1$, ρ_1 attains an asymptotically optimal forwarding index if $n\to\infty$.

[Theorem 2]

$$\xi(G_B(n, d), \rho_1)<n\log_d n \quad \text{if } \gcd(n, d)=1. \qquad \square$$

To prove Theorem 2, the following property is used.

[Property 5][12] The number of solution x of the congruence equation,

$$p\cdot x\equiv q \pmod n$$

is $\gcd(p, n)$, if and only if q is a multiple of $\gcd(p, n)$. $\square$

(Proof of Theorem 2) Let the d-ary D-digit representation of n be $(r_1, r_2, \cdots, r_D)$, that is $n=\sum_{j=1}^{D} r_j\cdot d^{D-j}$ and an arbitrary node be v. Let us count the number of routes which pass through v. For any other node u, the routes from u to any node in V defined by ρ_1 is represented by d-ary D-digits less that n, that is, $(0, 0, \cdots, 0)$, $(0, 0, \cdots, 1)$, $\cdots$, $(r_1, r_2, \cdots, r_D-1)$. Consider the equation

$$v\equiv d^i\cdot u+y \pmod n. \qquad (9)$$

If $0\leqq y<d^i$ and d-ary i-digit representation of y is $(y_1, y_2, \cdots, y_i)$, there is a walk from u to v represented by $(y_1, y_2, \cdots, y_i)$, and there are d^{D-i} D-length walks from u to some nodes in V via v, which are represented by $(y_1, y_2, \cdots, y_i, q_1, q_2, \cdots, q_{D-i})(0\leqq q_i<d)$. Thus, the number of walks from u to V defined by ρ_1 whose i-th intermediate node is v is:

$$\begin{cases} d^{D-i} & \text{if } 0\leqq y<\sum_{j=1}^{i} r_j\cdot d^{i-j} \\ \sum_{j=i+1}^{D} r_j\cdot d^{D-j} & \text{if } y=\sum_{j=1}^{i} r_j\cdot d^{i-j} \\ 0 & \text{otherwise} \end{cases}$$

From Property 5, Eq. (9) has one solution u for each integer $y(0\leqq y<n)$, because $\gcd(n, d)=1$. Thus, the number of routes which have v as the i-th intermediate node is

$$d^{D-i}\cdot\sum_{j=1}^{i} r_j\cdot d^{i-j}+\sum_{j=i+1}^{D} r_j\cdot d^{D-j}=n.$$

Since i is $1, 2, \cdots, D-1$, the number of routes which pass through v is $(D-1)n$. Therefore,

$$\xi_v(G_B(n, d), \rho_1)\leqq(D-1)n<n\log_d n$$

and

$$\xi(G_B(n, d), \rho_1) < n \log_d n. \qquad \square$$

Next, let us consider general cases containing the case of $\gcd(n, d) \neq 1$. Let $t = \lfloor d^D/n \rfloor$. Thus, $tn \leqq d^D < (t+1)n$ and $1 \leqq t < d$. Now another routing ρ_2 is defined.

[Definition 3]

$$\rho_2(u, v) = W_{D,j}(u, v),$$

where $j \equiv P_D(u, v) \pmod{t}$. $\square$

[Theorem 3]

$$\xi(G_B(n, d), \rho_2) < 2n \lceil \log_d n \rceil. \qquad \square$$

(Proof) Let the d-ary D-digit representation of n be $(r_1, r_2, \cdots, r_D)$ and an arbitrary node be v. Let us count the number of routes which pass through v. For another node u, of the d^D D-length walks from u to V, n walks represented by $(0, 0, \cdots, 0)$, $(0, 0, \cdots, 1)$, $\cdots$, $(r_1, r_2, \cdots, r_D - 1)$ are walks of $W_{D,0}(u)$, $(r_1, r_2, \cdots, r_D)$, $(r_1, r_2, \cdots, r_D + 1)$, $\cdots$, (d-ary D-digit representation of $2n-1$) are walks of $W_{D,1}(u)$, and so on. From the definition of ρ_2, if a walk in $W_{D,j}(u)$ is used as the route from u to a node w, walks in $W_{D,j}(u)$ are not used as routes from u to $w+1, w+2, \cdots, w+t-1$ and used again from u to $w+t$ if $W_{D,j}(u, w+t)$ is defined. Thus, every t-th walk in $W_{D,j}(u)$ is used as ρ_2. Consider Eq. (9). If $0 \leqq y < d^i$ and the d-ary i-digit representation of y is $(y_1, y_2, \cdots, y_i)$, there are d^{D-i} D-length walks from u to V via v, which are represented by $(y_1, y_2, \cdots, y_i, q_1, q_2, \cdots, q_{D-i})(0 \leqq q_i < d)$. Let j_0 be the smallest integer j such that there is at least one walk of $W_{D,j}(u)$ in these d^{D-i} walks. For $j' > j_0 + 1$, there can not be a walk of $W_{D,j'}(u)$ in these d^{D-i} walks because $|W_{D,j}(u)| = n > d^{D-i}$. Let a of these d^{D-i} walks be in $W_{D,j}(u)$ and $d^{D-i} - a$ be in $W_{D,j+1}(u)$. Thus, at most

$$\left\lceil \frac{a}{t} \right\rceil + \left\lceil \frac{d^{D-i} - a}{t} \right\rceil$$

walks are used as routes from u. From Property 5, the number of solution u for Eq. (9) is $\gcd(d^i, n)$, if $v - y$ is a multiple of $\gcd(d^i, n)$. The sum of the numbers of solution u for these d^i equations for different $y (0 \leqq y < d^i)$ is equal to d^i, because every $\gcd(d^i, n)$-th equation of these d^i equations has $\gcd(d^i, n)$ solutions. Thus,

$$\begin{aligned}
&\xi_v(G_B(n, d), \rho_2) \\
&\leqq \sum_{i=1}^{D-1} d^i \left(\left\lceil \frac{a}{t} \right\rceil + \left\lceil \frac{d^{D-i} - a}{t} \right\rceil \right) \\
&\leqq \sum_{i=1}^{D-1} d^i \left(\frac{d^{D-i}}{t} + \frac{2(t-1)}{t} \right) \\
&= \frac{(D-1)d^D}{t} + \frac{2(t-1)(d^D - d)}{(d-1)t} \\
&< \frac{t+1}{t}(D-1)n + \frac{2(t^2-1)}{t^2} n < 2nD
\end{aligned}$$

because

$$\left\lceil \frac{b}{a} \right\rceil \leqq \frac{b}{a} + \frac{a-1}{a},$$

$1 \leqq t < d$ and $d^D < (t+1)n$. Therefore,

$$\xi_v(G_B(n, d), \rho_2) < 2n \lceil \log_d n \rceil$$

and

$$\xi(G_B(n, d), \rho_2) < 2n \lceil \log_d n \rceil. \qquad \square$$

Theorem 3 can derive a method of constructing an undirected graph and routing which attains a quasi-optimal forwarding index. Let $G_B'(n, d)$ be the undirected graph modifying directed edges of $G_B(n, d)$ into undirected ones. Note that $\Delta(G_B'(n, d)) = 2d$. Therefore, for any n and d, an n-node undirected graph with $\Delta = d$, $G_u(n, d)$ can be obtained from $G_B'(n, \lfloor d/2 \rfloor)$ by adding some edges if d is odd. If ρ_2 is used as a routing,

$$\xi(G_u(n, d), \rho_2) = \xi(G_B(n, \lfloor d/2 \rfloor), \rho_2) < 2n \lceil \log_{\lfloor d/2 \rfloor} n \rceil.$$

A previous paper[(3)] has shown a construction method in which

$$\xi(G(n, d), \rho) < 3n \log_{\lfloor d/3 \rfloor} n.$$

Since $2n \lceil \log_{\lfloor d/2 \rfloor} n \rceil < 3n \log_{\lfloor d/3 \rfloor} n$ if $n \geqq \lfloor d/2 \rfloor \cdot \lfloor d/3 \rfloor$, the forwarding index of this construction method is smaller than that of the previously proposed one when $n \geqq \lfloor d/2 \rfloor \cdot \lfloor d/3 \rfloor$.

5. Construction of a Digraph with a Reliable and Efficient Routing

This section considers a construction method for a digraph and routing which have both a quasi-minimal forwarding index and a quassi-minimal surviving route graph diameter. As mentioned in Sect. 4, the generalized de Bruijn digraph $G_B(n, d)$ has quasi-maximal connectivity. In the case of $\gcd(n, d) = 1$, a routing ρ_1' on $G_B(n, d)$ is derived by modifying the routing ρ_1 defined in Definition 2.

[Definition 4]

$$\rho_1'(u, v) = \begin{cases} W_{D-1}(u, v) & \text{if} \quad P_{D-1}(u, v) < d^{D-1} \\ W_{D,0}(u, v) & \text{otherwise} \end{cases}$$

[Theorem 4]

$$\xi(G_B(n, d), \rho_1') < 2n \log_d n$$

$$\max_{|F| < d-1} D(R(G_B(n, d), \rho_1')/F) \leqq 3$$

if $\gcd(n, d) = 1$ and $n > d^4$. $\square$

(Proof)

(Forwarding index) For any node v, let us count the number of routes in W_{D-1} which pass through v. For a node pair u and w such that $P_{D-1}(u, w) < d^{D-1}$, the routes from u to w defined by $W_{D-1}(u)$ are represented by d-ary $(D-1)$-digits $(q_1, q_2, \cdots, q_{D-1})(0 \leqq q_i < d)$. As in the proof of Theorem 2, let us consider Eq. (9). The number of routes of $W_{D-1}(u)$ in which v appears as the i-th intermediate node is :

$$\begin{cases} d^{D-i-1} & \text{if } 0 \leqq y < d^i \\ 0 & \text{otherwise} \end{cases}$$

Since $\gcd(n, d)=1$, the number of solution u of Eq. (9) is 1 for any integer y. Thus, the number of routes of W_{D-1} which have v as the i-th intermediate node is $d^{D-i-1} \cdot d^i = d^{D-1}$. Since i is $1, 2, \cdots, D-2$, the number of routes of W_{D-1} which pass through v is at most $(D-2) \cdot d^{D-1}$.

Therefore,

$$\xi_v(G_B(n, d), \rho_1') \leqq \xi_v(G_B(n, d), \rho_1) + (D-2) \cdot d^{D-1}$$
$$= (D-1) \cdot d^D + (D-2) \cdot d^{D-1} < 2n \log_d n$$

and

$$\xi(G_B(n, d), \rho_1') < 2n \log_d n.$$

(Diameter of surviving route graph) Assume two distinct nodes in $V-F$, u and v. Let n_1 of the n routes from u to V be surviving, n_2 of the n routes from V to v be surviving, and f_3 of n^2 routes from V to V be non-surviving. If $n_1>0$, $n_2>0$ and $n_1 \cdot n_2 > f_3$, there are two nodes u' and v' such that the routes from u to u', from u' to v', and from v' to v are all surviving. Thus, $D(R(G_B(n, d), \rho_1')/F) \leqq 3$.

From now on, only node faults are considered. If an edge fault occurs, it is considered as a node fault adjacent to the edge.

First consider routes from u to V. Let us count the number of surviving routes of d^{D-1} routes of $W_{D-1}(u)$. Let f be a faulty node. Similar to the proof of Theorem 2, if u and f satisfy the following equation for some $i (1 \leqq i \leqq D-1)$,

$$f \equiv u \cdot d^i + y \pmod n \quad 0 \leqq y < d^i, \tag{10}$$

d^{D-i-1} walks pass through or terminate at f, that is, these walks are not surviving. Let μ_i be the number of faulty nodes in F which satisfy Eq. (10) for $i (1 \leqq i \leqq D-1)$. Thus, the number of surviving routes from u to V is

$$n_1 \geqq d^{D-1} - \sum_{i=1}^{D-1} \mu_i \cdot d^{D-i-1}.$$

Next let us consider n_2. Similar to the above discussion, if u and $f \in F$ satisfy

$$v \equiv f \cdot d^i + y \pmod n \quad 0 \leqq y < d^i, \tag{11}$$

then d^{D-i-1} walks pass through or initiate at f, that is, these walks are not surviving. Let ν_i be the number of faulty nodes in F which satisfies Eq. (11) for i $(1 \leqq i \leqq D-1)$. Thus, the number of surviving routes from u to V is

$$n_2 \geqq d^{D-1} - \sum_{i=1}^{D-1} \nu_i \cdot d^{D-i-1}.$$

Now let us prove if $i_1 + i_2 \leqq D-1$, $\mu_{i_1} + \nu_{i_2} \leqq |F|+1$. Assume $\mu_{i_1} + \nu_{i_2} > |F|+1$. Then, there are two distinct nodes $f_1, f_2 \in F$ satisfy Eq. (10) for $i=i_1$ and Eq. (11) for $i=i_2$, that is,

$$f_k \equiv u d^{i_1} + y_k \pmod n, \; v \equiv f_k d^{i_2} + y_k' \pmod n$$
$$0 \leqq y_k < d^{i_1}, \; 0 \leqq y_k' < d^{i_2} \quad (k=1, 2).$$

From there equation, there is an integer $j (\neq 0)$ which satisfies

$$(y_1 - y_2) d^{i_2} + (y_1' - y_2') = jn.$$

On the other hand, if $i_1 + i_2 \leqq D-1$,

$$|(y_1 - y_2) d^{i_2} + (y_1' - y_2')| \leqq d^{D-1} - 1 < n,$$

because $0 \leqq y_k < d^{i_1}$, $0 \leqq y_k' < d^{i_2}$. Thus, if $i_1 + i_2 \leqq D-1$, it is deduced $f_1 = f_2$, that is, $\mu_{i_1} + \nu_{i_2} \leqq |F|+1$.

Since $\mu_i, \nu_i \leqq |F| \leqq d-2$ for $i=3, \cdots, D-1$,

$$n_1 \cdot n_2 \geqq \left(d^{D-1} - \mu_1 d^{D-2} - \mu_2 d^{D-3} - \sum_{i=3}^{D-1} (d-2) d^{D-i-1} \right) \cdot \left(d^{D-1} - \nu_1 d^{D-2} - \nu_2 d^{D-3} - \sum_{i=3}^{D-1} (d-2) \cdot d^{D-i-1} \right). \tag{12}$$

Since $n > d^4$, that is, $D \geqq 5$, $\mu_{i_1} + \nu_{i_2} \leqq d-1$ for $i_1 + i_2 \leqq 4$. From these inequalities and $0 \leqq \mu_i, \nu_i \leqq d-2 (i=1, 2)$, the right side of inequality (12) is minimized when $\mu_1 = 1$, $\nu_1 = d-2$, $\mu_2 = 1$ and $\nu_2 = d-2$ (or $\mu_1 = d-2$, $\nu_1 = 1$, $\mu_2 = d-2$ and $\nu_2 = 1$). Therefore,

$$n_1 \cdot n_2 > (d-2)(d+1)(d^{2D-5} + d^{2D-6}).$$

Next, consider f_3. Let f be a faulty node. For any node w, consider the equation

$$f \equiv w \cdot d^i + y \pmod n, \quad 0 \leqq y < d^i. \tag{13}$$

The sum of the number of solutions w for all y is d^i as in the proof of Theorem 2. For each w, the number of walks which pass through f is at most $d^{D-i} + d^{D-i-1}$, where d^{D-i} is for $W_D(w)$ and d^{D-i-1} is for $W_{D-1}(w)$. Therefore,

$$f_3 = (d-2) \cdot \left(\sum_{i=1}^{D-1} d^i \cdot d^{D-i} + \sum_{i=1}^{D-2} d^i \cdot d^{D-i-1} \right)$$
$$= (d-2) \cdot ((D-1) \cdot d^D + (D+2) \cdot d^{D-1}),$$

and if $D \geqq 5$ and $d \geqq 3$,

$$n_1 \cdot n_2 - f_3 > (d-2)(d+1)(d^{2D-5} + d^{2D-6})$$
$$-(d-2) \cdot ((D-1) \cdot d^D + (D-2) \cdot d^{D-1}) > 0$$

and

$$\max_{|F|<d-1} D(R(G_B(n, d), \rho_1')/F) \leqq 3. \qquad \square$$

For the case of $\gcd(n, d) \neq 1$, modify the routing ρ_2 as follows.

[Definition 5]

$$\rho_2'(u, v) = \begin{cases} W_{D-1}(u, v) & \text{if } P_{D-1}(u, v) < d^{D-1} \\ W_{D,j}(u, v) & \text{otherwise} \end{cases}$$

where $j \equiv P_D(u, v) \pmod t$. □

[Theorem 5]

$$\xi(G_B(n, d), \rho_2') < 3n \log_d n$$

$$\max_{|F|<d-1} D(R(G_B(n, d), \rho_2')/F \leqq 3$$

if $n > d^4$ and $d \geqq 3$. □

(Proof)

(Forwarding index) Let us consider the number of routes is W_{D-1} which pass through a node v. As in the proof of Theorem 3, since Eq. (9) has $\gcd(d^i, n)$ solutions if $v-y$ is a multiple of $\gcd(d^i, n)$, the sum of the number of solutions is d^i. Thus, as in the proof of Theorem 4, the number of routes in W_{D-1} which pass through v is at most $(D-2) \cdot d^{D-1}$. Therefore,

$$\begin{aligned} \xi_v(G_B(n, d), \rho_2') &\leqq \xi_v(G_B(n, d), \rho_2) + (D-2) \cdot d^{D-1} \\ &\leqq \frac{(D-1)d^D}{t} + \frac{2(d^D-d)(t-1)}{(d-1)t} + (D-2)d^{D-1} \\ &< \frac{(D-1)(t+1)n}{t} + \frac{2(t+1)n-2d}{d-1} \\ &\quad + \frac{(D-2)(t+1)n}{d} \\ &< 3n(D-1) < 3n \log_d n \end{aligned}$$

because $d \geqq 3, 1 \leqq t < d, d^D < (t+1)n$ and $n > d^4$ that is, $D \geqq 5$.

The proof of the surviving route graph is the same at that of Theorem 4. □

6. Conclusion

This paper presents (1) a sufficient condition for k-connected digraphs to have a routing whose surviving route graph diameter is a small constant, (2) a construction of a digraph and routing with quasi-minimal forwarding index for any number of nodes n and maximum degree d, (3) a construction of a digraph and routing with quasi-minimal forwarding index and a quasiminimal surviving route graph diameter for any number of nodes n and maximum degree d.

The following problems remain for further study: (1) a sufficient condition for k-connected digraphs to have a routing such that the diameter of surviving route graph is 2, (2) construction of a digraph and routing whose forwarding index is equal to the lower bound for any number of nodes n and maximum degree d. (3) construction of a maximally connected digraph and routing with quasi-minimal forwarding index and quasi-minimal surviving route graph diameter for any number of nodes n and maximum degree d. Routings on maximally connected quasi-minimal diameter digraphs for any number of nodes n and maximum degree d such as shown in Footnote (see p. 1212) should be considered.

References

(1) B. Bollobas: "Extremal Graph Theory", Academic Press, London (1978).

(2) A. Broder, D. Dolev, M. Fischer and B. Simons: "Efficient fault tolerant routing in networks", Proc. of ACM 16th STOC, pp. 536-541 (May 1984).

(3) F. R. K. Chung, E. D. Coffman, M. I. Reiman and B. Simon: "The forwarding index of communication networks", IEEE Trans. Inf. Theory, **IT-33**, 2, pp. 224-233 (March 1987).

(4) D. Dolev, J. Y. Halpern, B. Simons and H. R. Strong: "A new look at fault tolerant routing", Proc. of ACM 16th STOC, pp. 526-535 (May 1984).

(5) F. Harary: "Graph Theory", Addison-Wesley, Reading, MA (1969).

(6) M. Imase and M. Itoh: "Design to minimize diameter on building-block network", IEEE Trans. Comput., **C-30**, 6, pp. 439-442 (June 1981).

(7) M. Imase and Y. Manabe: "Fault tolerant routings in a κ-connected network", Information Processing Letters, **28**, 4, pp. 171-175 (July 1988).

(8) M. Imase, T. Soneoka and K. Okada: "Connectivity of regular directed graphs with small diameter", IEEE Trans. Comput., **C-34**, 3, pp. 267-273 (March 1985).

(9) K. Kawaguchi and K. Wada: "Optimal fault-tolerant network routings for $(k+1)$-connected graphs", Univ. of Wiscosin-Milwaukee Commputer Science Tech. Rep. UWM EE&CS 88-2 (March 1988).

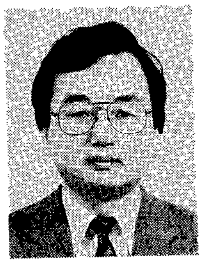
(10) Y. Manabe, M. Imase and T. Soneoka: "Reliability and efficiency of networks with fixed routings", 1988 Spring IEICE Natl. Conv. Rec., SD-5-9 (March 1988).

(11) D. Peleg and B. Simons: "On fault tolerant routing in general graphs", Proc. of 5th Principles of Distributed Computing, pp. 98-107 (Aug. 1986).

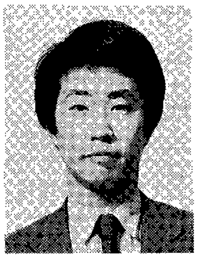
(12) B. L. van der Waerden: "Algebra", pp. 63-65, Springer-Verlag, Berlin (1964).

(13) R. S. Wilcov: "Analysis and design of reliable computer networks", IEEE Trans. Commun., **COM-20**, 6, pp. 660-678 (June 1972).

Yoshifumi Manabe was born in Osaka, Japan, on December 7, 1960. He received his B. E. and M. E. degrees in information engineering from Onaka University in 1983 and 1985, respectively. In 1985 he joined NTT Electrical Communications Laboratories (ECL). Since then, he has been engaged in research on communications networks and distributed algorithms. He is now a Research Enginner in the NTT Software Laboratories. Mr. Manabe is a member of ACM and IPSJ.



Makoto Imase was born in Osaka, Japan, on April 14, 1952. He received his B. E. and M. E. degrees in information engineering from Osaka University in 1975 and 1977, respectively. He received his D. E. degree from Osaka University in 1986. In 1977 he joined Electrical Communications Laboratories, Nippon Telegraph and Telephone Corporation (NTT), where he has been engaged in research on distributed switching systems and communications networks. He is now a Senior Research Engineer, Supervisor in the NTT Software Laboratories. Dr. Imase is a member of the IEEE Computer Society and received the IEICE Young Engineer Award in 1984.



Terunao Soneoka was born in Tokyo, Japan on June 15, 1957. He received the B. E. and M. E. degrees in electrical engineering from the University of Tokyo, Tokyo, Japan in 1980 and 1982 respectively. In 1982 he joined NTT Electrical Communications Laboratories (ECL). Since then, he has been engaged in the fundamental research of telecommunications network and fault-tolerant communications software. He is now a Research Engineer in the NTT Software Laboratories. Mr. Soneoka is a member of IEEE and ACM.